18.031 Laplace Transform Table

Properties and Rules

Function	**Transform**	
$f(t)$	$F(s) = \int_{0^-}^{\infty} f(t)e^{-st}\,dt$	(Definition)
$a\,f(t) + b\,g(t)$	$a\,F(s) + b\,G(s)$	(Linearity)
$e^{at}f(t)$	$F(s-a)$	(s-shift)
$f'(t)$	$sF(s) - f(0^-)$	
$f''(t)$	$s^2F(s) - sf(0^-) - f'(0^-)$	
$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0^-) - \quad \cdots - f^{(n-1)}(0^-)$	
$tf(t)$	$-F'(s)$	
$t^nf(t)$	$(-1)^nF^{(n)}(s)$	
$u(t-a)f(t-a)$	$e^{-as}F(s)$	(t-translation or t-shift)
$u(t-a)f(t)$	$e^{-as}\mathcal{L}(f(t+a))$	(t-translation)
$(f*g)(t) = \int_{0^-}^{t^+} f(t-\tau)\,g(\tau)\,d\tau$	$F(s)\,G(s)$	
$\int_{0^-}^{t^+} f(\tau)\,d\tau$	$\frac{F(s)}{s}$	(integration rule)

Interesting, but not included in this course.

$\frac{f(t)}{t}$	$\int_s^{\infty} F(\sigma)\,d\sigma$	

(The function table is on the next page.)

Function Table

Function	**Transform**	**Region of convergence**
1	$1/s$	$\text{Re}(s) > 0$
e^{at}	$1/(s-a)$	$\text{Re}(s) > \text{Re}(a)$
t	$1/s^2$	$\text{Re}(s) > 0$
t^n	$n!/s^{n+1}$	$\text{Re}(s) > 0$
$\cos(\omega t)$	$s/(s^2+\omega^2)$	$\text{Re}(s) > 0$
$\sin(\omega t)$	$\omega/(s^2+\omega^2)$	$\text{Re}(s) > 0$
$e^{at}\cos(\omega t)$	$(s-a)/((s-a)^2+\omega^2)$	$\text{Re}(s) > \text{Re}(a)$
$e^{at}\sin(\omega t)$	$\omega/((s-a)^2+\omega^2)$	$\text{Re}(s) > \text{Re}(a)$
$\delta(t)$	1	all s
$\delta(t-a)$	e^{-as}	all s
$\cosh(kt) = \dfrac{e^{kt}+e^{-kt}}{2}$	$s/(s^2-k^2)$	$\text{Re}(s) > k$
$\sinh(kt) = \dfrac{e^{kt}-e^{-kt}}{2}$	$k/(s^2-k^2)$	$\text{Re}(s) > k$
$\dfrac{1}{2\omega^3}(\sin(\omega t) - \omega t\cos(\omega t))$	$\dfrac{1}{(s^2+\omega^2)^2}$	$\text{Re}(s) > 0$
$\dfrac{t}{2\omega}\sin(\omega t)$	$\dfrac{s}{(s^2+\omega^2)^2}$	$\text{Re}(s) > 0$
$\dfrac{1}{2\omega}(\sin(\omega t) + \omega t\cos(\omega t))$	$\dfrac{s^2}{(s^2+\omega^2)^2}$	$\text{Re}(s) > 0$
$u(t-a)$	e^{-as}/s	$\text{Re}(s) > 0$
$t^n e^{at}$	$n!/(s-a)^{n+1}$	$\text{Re}(s) > \text{Re}(a)$

Interesting, but not included in this course.

$\dfrac{1}{\sqrt{\pi t}}$	$\dfrac{1}{\sqrt{s}}$	$\text{Re}(s) > 0$
t^a	$\dfrac{\Gamma(a+1)}{s^{a+1}}$	$\text{Re}(s) > 0$