

MIT Integration Bee: Semifinals
(Time limit per integral: 4 minutes)

Semifinal #1 Problem 1

$$\lim_{n \rightarrow \infty} \int_0^{\infty} \sqrt{n} \left(\frac{e^{x-1}}{x^x} \right)^n dx$$

Semifinal #1 Problem 1

$$\lim_{n \rightarrow \infty} \int_0^\infty \sqrt{n} \left(\frac{e^x - 1}{x^x} \right)^n dx = \boxed{\sqrt{2\pi}}$$

Semifinal #1 Problem 2

$$\int \frac{x^2 - 2}{(x^2 + 2)\sqrt{x^4 + 4}} dx$$

Semifinal #1 Problem 2

$$\int \frac{x^2 - 2}{(x^2 + 2)\sqrt{x^4 + 4}} dx = \frac{1}{2} \arctan \left(\frac{\sqrt{x^4 + 4}}{2x} \right)$$

Semifinal #1 Problem 3

$$\int_0^\pi \min_{a \in \{0,1\}} \max_{b \in \{0,1\}} \min_{c \in \{0,1\}} \cos((a+b+c)x) dx$$

Semifinal #1 Problem 3

$$\int_0^\pi \min_{a \in \{0,1\}} \max_{b \in \{0,1\}} \min_{c \in \{0,1\}} \cos((a+b+c)x) dx$$
$$= \boxed{-\frac{3\sqrt{3}}{4}}$$

Semifinal #1 Problem 4

$$\left[\int_0^2 100 \left(\sqrt{-4x^2 + 8x + 16} - \sqrt{-x^2 - 2x + 9} \right) dx \right]$$

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$$\left[\int_0^2 100 \left(\sqrt{-4x^2 + 8x + 16} - \sqrt{-x^2 - 2x + 9} \right) dx \right]$$
$$= \boxed{400}$$

MIT Integration Bee: Semifinal Tiebreakers

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Semifinal Tiebreakers Problem 1

$$\int \left(1 + \frac{1}{x}\right) \left(1 - \frac{2}{x}\right) x^{\sqrt{3}} e^x dx$$

Semifinal Tiebreakers Problem 1

$$\int \left(1 + \frac{1}{x}\right) \left(1 - \frac{2}{x}\right) x^{\sqrt{3}} e^x dx$$
$$= \boxed{\left(x^{\sqrt{3}} - (\sqrt{3} + 1)x^{\sqrt{3}-1}\right) e^x}$$

Semifinal #2 Problem 1

$$\lim_{n \rightarrow \infty} \int_{-2}^2 \underbrace{\left(\left(\cdots \left((x^2 - 2)^2 - 2 \right)^2 \cdots - 2 \right)^2 \right)^2}_{n \text{ squares}} dx$$

Semifinal #2 Problem 1

$$\lim_{n \rightarrow \infty} \int_{-2}^2 \underbrace{\left(\left(\dots \left((x^2 - 2)^2 - 2 \right)^2 \dots - 2 \right)^2 \right)^2}_{n \text{ squares}} dx = \boxed{24}$$

Semifinal #2 Problem 2

$$\int_{-\infty}^{\infty} \left[\sqrt{\left| \frac{(x-1)(x+1)}{(x-2)x(x+2)} \right|} \right] dx$$

Semifinal #2 Problem 2

$$\int_{-\infty}^{\infty} \left[\sqrt{\left| \frac{(x-1)(x+1)}{(x-2)x(x+2)} \right|} \right] dx = \boxed{\frac{\pi^2}{3}}$$

Semifinal #2 Problem 3

$$\int_0^\pi \left(\frac{\cos(2x) \cos(3x) \cos(5x) \cos(7x) \cos(11x)}{\cos(13x) \cos(17x) \cos(19x) \cos(23x)} \right) dx$$

Semifinal #2 Problem 3

$$\int_0^\pi \left(\cos(2x) \cos(3x) \cos(5x) \cos(7x) \cos(11x) \right. \\ \left. \cdot \cos(13x) \cos(17x) \cos(19x) \cos(23x) \right) dx \\ = \boxed{\frac{5\pi}{256}}$$

Semifinal #2 Problem 4

$$\int \frac{dx}{x^4 + 5x^3 + 10x^2 + 9x + 3}$$

Semifinal #2 Problem 4

$$\int \frac{dx}{x^4 + 5x^3 + 10x^2 + 9x + 3}$$
$$= \boxed{\frac{1}{2} \log(x^2 + 3x + 3) - \log(x + 1) - \frac{1}{x + 1} - \frac{1}{\sqrt{3}} \arctan\left(\frac{2x + 3}{\sqrt{3}}\right)}$$