

MIT Integration Bee: Regular Season

(Time limit per integral: 2 minutes)

Regular Season Problem 1

$$\int_0^{2026} \left(\sum_{k=0}^{\infty} \frac{x^{2026k+2025}}{k!} \right) dx$$

Regular Season Problem 1

$$\int_0^{2026} \left(\sum_{k=0}^{\infty} \frac{x^{2026k+2025}}{k!} \right) dx = \boxed{\frac{e^{2026^{2026}} - 1}{2026}}$$

Regular Season Problem 2

$$\int_0^{\log(34)} \left(2 + \frac{1}{2e^x - 1} + \frac{1}{3e^x - 1} \right) dx$$

Regular Season Problem 2

$$\int_0^{\log(34)} \left(2 + \frac{1}{2e^x - 1} + \frac{1}{3e^x - 1} \right) dx = \boxed{\log \left(\frac{6767}{2} \right)}$$

Regular Season Problem 3

$$\int_0^{1/2} \left(\cos(\pi x) - \pi \left(\frac{1}{4} - x^2 \right) \left(\frac{5}{4} - x^2 \right) \right) dx$$

Regular Season Problem 3

$$\int_0^{1/2} \left(\cos(\pi x) - \pi \left(\frac{1}{4} - x^2 \right) \left(\frac{5}{4} - x^2 \right) \right) dx$$
$$= \boxed{\frac{1}{\pi} - \frac{\pi}{10}}$$

Regular Season Problem 4

$$\int_{\int_0^1 \int_0^1 x \, dx}^{\int_0^1 \int_0^1 x \, dx} x \, dx$$

Regular Season Problem 4

$$\int_0^1 \int_0^1 \int_0^1 \int_0^1 \int_0^1 \int_0^1 x \, dx \, dx \, dx \, dx \, dx = \boxed{-\frac{189}{2048}}$$

Regular Season Problem 5

$$\int \frac{4x^2 - 1}{x^2(x^2 - 1)} dx$$

Regular Season Problem 5

$$\int \frac{4x^2 - 1}{x^2(x^2 - 1)} dx = \boxed{-\frac{1}{x} + \frac{3}{2} \log \left(\frac{x - 1}{x + 1} \right)}$$

Regular Season Problem 6

$$\int \frac{dx}{27x - x^{-1/3}}$$

Regular Season Problem 6

$$\int \frac{dx}{27x - x^{-1/3}} = \boxed{\frac{1}{36} \log(27x^{4/3} - 1)}$$

Regular Season Problem 7

$$\int \frac{2\sqrt{x} + 1}{\sqrt{x^2 + x\sqrt{x}}} dx$$

Regular Season Problem 7

$$\int \frac{2\sqrt{x} + 1}{\sqrt{x^2 + x\sqrt{x}}} dx = \boxed{4\sqrt{x + \sqrt{x}}}$$

Regular Season Problem 8

$$\int \frac{e^x}{e^{2x} - e^{-2x}} dx$$

Regular Season Problem 8

$$\int \frac{e^x}{e^{2x} - e^{-2x}} dx = \boxed{\frac{1}{2}(\arctan(e^x) - \operatorname{arctanh}(e^x))}$$

Regular Season Problem 9

$$\int_0^\pi \frac{|\sin(x) + \sin(2x) + \sin(3x) + \sin(4x)|}{\sin(x) + \sin(2x) + \sin(3x) + \sin(4x)} dx$$

Regular Season Problem 9

$$\int_0^\pi \frac{|\sin(x) + \sin(2x) + \sin(3x) + \sin(4x)|}{\sin(x) + \sin(2x) + \sin(3x) + \sin(4x)} dx$$
$$= \boxed{\frac{2\pi}{5}}$$

Regular Season Problem 10

$$\int_0^1 \frac{1-x}{\sqrt{e^{2x}-x^2}} dx$$

Regular Season Problem 10

$$\int_0^1 \frac{1-x}{\sqrt{e^{2x}-x^2}} dx = \boxed{\arcsin\left(\frac{1}{e}\right)}$$

Regular Season Problem 11

$$\int_0^{2026} \left\{ x + \frac{1}{2} \left\lfloor \frac{x}{2} \right\rfloor + \frac{1}{3} \left\lfloor \frac{x}{3} \right\rfloor + \frac{1}{4} \left\lfloor \frac{x}{4} \right\rfloor + \cdots \right\} dx$$

Regular Season Problem 11

$$\int_0^{2026} \left\{ x + \frac{1}{2} \left\lfloor \frac{x}{2} \right\rfloor + \frac{1}{3} \left\lfloor \frac{x}{3} \right\rfloor + \frac{1}{4} \left\lfloor \frac{x}{4} \right\rfloor + \cdots \right\} dx$$
$$= \boxed{1013}$$

Regular Season Problem 12

$$\int_{1/2}^2 \frac{x^8}{x^8 - x^6 + x^4 - x^2 + 1} dx$$

Regular Season Problem 12

$$\int_{1/2}^2 \frac{x^8}{x^8 - x^6 + x^4 - x^2 + 1} dx = \boxed{\frac{3}{2}}$$

Regular Season Problem 13

$$\int (\operatorname{arcsinh}(x))^2 dx$$

Regular Season Problem 13

$$\int (\operatorname{arcsinh}(x))^2 dx$$
$$= \boxed{x(\operatorname{arcsinh}(x))^2 - 2\operatorname{arcsinh}(x)\sqrt{1+x^2} + 2x}$$

Regular Season Problem 14

$$\int \cos^2(2026x) \cos(1013x) \, dx$$

Regular Season Problem 14

$$\int \cos^2(2026x) \cos(1013x) dx$$
$$= \boxed{\frac{\sin(1013x)}{2026} + \frac{\sin(3039x)}{12156} + \frac{\sin(5065x)}{20260}}$$

Regular Season Problem 15

$$\int e^x \arcsin(\tanh(x)) \, dx$$

Regular Season Problem 15

$$\int e^x \arcsin(\tanh(x)) dx$$
$$= \boxed{e^x \arcsin(\tanh(x)) - \log(e^{2x} + 1)}$$

Regular Season Problem 16

$$\int \frac{x((1 - x^2) \cos(x) + 2x \sin(x))}{(1 + x^2)^2} dx$$

Regular Season Problem 16

$$\int \frac{x((1-x^2)\cos(x) + 2x\sin(x))}{(1+x^2)^2} dx$$
$$= \boxed{-\frac{\cos(x) + x\sin(x)}{1+x^2}}$$

Regular Season Problem 17

$$\int_0^1 \sin((x-1)(5x-1)) \sin(x(3x-2)) \, dx$$

Regular Season Problem 17

$$\int_0^1 \sin((x-1)(5x-1)) \sin(x(3x-2)) dx = \boxed{0}$$

Regular Season Problem 18

$$\int_{1/4}^{\sqrt{2}} \frac{dx}{x(x^{x^{x^{\cdots}}} \log(x) - 1)}$$

Regular Season Problem 18

$$\int_{1/4}^{\sqrt{2}} \frac{dx}{x(x^{x^{x^{\cdots}}} \log(x) - 1)} = \boxed{-\frac{3}{2}}$$

Regular Season Problem 19

$$\int_0^1 \frac{1 + ((2026 + 2x - x^2)^2 - 2)(2026 + 4x - 4x^2)^2}{(2025 + 2x - x^2)(2027 + 2x - x^2)(2025 + 4x - 4x^2)(2027 + 4x - 4x^2)} dx$$

Regular Season Problem 19

$$\int_0^1 \frac{1 + ((2026 + 2x - x^2)^2 - 2)(2026 + 4x - 4x^2)^2}{(2025 + 2x - x^2)(2027 + 2x - x^2)(2025 + 4x - 4x^2)(2027 + 4x - 4x^2)} dx = \boxed{1}$$

Regular Season Problem 20

$$\int_{1/2}^1 4^{x-1} \left(1 + 4^{4^{x-1}-1} \left(1 + 4^{4^{4^{x-1}-1}-1} (1 + \dots) \right) \right) dx$$

Regular Season Problem 20

$$\int_{1/2}^1 4^{x-1} \left(1 + 4^{4^{x-1}-1} \left(1 + 4^{4^{4^{x-1}-1}-1} (1 + \dots) \right) \right) dx$$
$$= \boxed{\frac{1}{2(\log(4) - 1)}}$$