

MIT Integration Bee: Quarterfinals

(Time limit per integral: 3 minutes)

Quarterfinal #1 Problem 1

$$\int \frac{\sin(x) + \cos(x)}{\sqrt{25 \sin^2(x) + 16 \cos^2(x)}} dx$$

Quarterfinal #1 Problem 1

$$\int \frac{\sin(x) + \cos(x)}{\sqrt{25 \sin^2(x) + 16 \cos^2(x)}} dx$$

$$= \frac{1}{3} \operatorname{arcsinh} \left(\frac{3}{4} \sin(x) \right) - \frac{1}{3} \arcsin \left(\frac{3}{5} \cos(x) \right)$$

Quarterfinal #1 Problem 2

$$\int_0^{\infty} \frac{x^3}{e^x + 1} dx$$

Quarterfinal #1 Problem 2

$$\int_0^{\infty} \frac{x^3}{e^x + 1} dx = \boxed{\frac{7\pi^4}{120}}$$

Quarterfinal #1 Problem 3

$$\int_0^1 \frac{1 - x^{5/2}}{1 - x} dx$$

Quarterfinal #1 Problem 3

$$\int_0^1 \frac{1 - x^{5/2}}{1 - x} dx = \boxed{\frac{46}{15} - 2 \log(2)}$$

Quarterfinal #2 Problem 1

$$\int_0^{\infty} \frac{dx}{x^6 + 1}$$

Quarterfinal #2 Problem 1

$$\int_0^{\infty} \frac{dx}{x^6 + 1} = \boxed{\frac{\pi}{3}}$$

Quarterfinal #2 Problem 2

$$\int_0^{\frac{1}{2}} \frac{\log(1-x)}{x} dx$$

Quarterfinal #2 Problem 2

$$\int_0^{\frac{1}{2}} \frac{\log(1-x)}{x} dx = \boxed{\frac{\log^2(2)}{2} - \frac{\pi^2}{12}}$$

Quarterfinal #3 Problem 1

$$\int \frac{\sqrt{x}}{1+x^2} dx$$

Quarterfinal #3 Problem 1

$$\int \frac{\sqrt{x}}{1+x^2} dx$$
$$= \frac{1}{\sqrt{2}} \left(\arctan \left(\frac{x-1}{\sqrt{2x}} \right) - \operatorname{arctanh} \left(\frac{\sqrt{2x}}{x+1} \right) \right)$$

Quarterfinal #3 Problem 2

$$\int_1^{\infty} \frac{(-1)^{\lfloor x \rfloor + \lfloor \frac{x}{2} \rfloor + \lfloor \frac{x}{3} \rfloor + \lfloor \frac{x}{4} \rfloor + \cdots}}{x^2} dx$$

Quarterfinal #3 Problem 2

$$\int_1^{\infty} \frac{(-1)^{\lfloor x \rfloor + \lfloor \frac{x}{2} \rfloor + \lfloor \frac{x}{3} \rfloor + \lfloor \frac{x}{4} \rfloor + \dots}}{x^2} dx = \boxed{1 - \frac{\pi^2}{6}}$$

Quarterfinal #3 Problem 3

$$\int_0^{\sqrt{3}} \arctan \left(\frac{3x - x^3}{1 - 3x^2} \right) dx$$

Quarterfinal #3 Problem 3

$$\int_0^{\sqrt{3}} \arctan \left(\frac{3x - x^3}{1 - 3x^2} \right) dx = \boxed{\frac{\pi}{\sqrt{3}} - 3 \log(2)}$$

MIT Integration Bee: Quarterfinal Tiebreakers

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Quarterfinal Tiebreakers Problem 1

$$\int_0^{\infty} e^{\frac{-x^5}{2025}} x^{\frac{3}{2}} dx$$

Quarterfinal Tiebreakers Problem 1

$$\int_0^\infty e^{\frac{-x^5}{2025}} x^{\frac{3}{2}} dx = \boxed{9\sqrt{\pi}}$$

Quarterfinal #4 Problem 1

$$\int_1^{\infty} \frac{dx}{[x]^2 \lceil x \rceil^2}$$

Quarterfinal #4 Problem 1

$$\int_1^{\infty} \frac{dx}{[x]^2 \lceil x \rceil^2} = \boxed{\frac{\pi^2 - 9}{3}}$$

Quarterfinal #4 Problem 2

$$\int_0^{\infty} \frac{4 \cos(4x)}{x^4 + 4} dx$$

Quarterfinal #4 Problem 2

$$\int_0^\infty \frac{4 \cos(4x)}{x^4 + 4} dx = \boxed{\frac{\pi}{2e^4}(\sin(4) + \cos(4))}$$

Quarterfinal #4 Problem 3

$$\int \sin(\operatorname{arccosh}(x)) \, dx$$

Quarterfinal #4 Problem 3

$$\int \sin(\operatorname{arccosh}(x)) \, dx$$

$$= \boxed{\frac{1}{2} \left(x \sin(\operatorname{arccosh}(x)) - \sqrt{x^2 - 1} \cos(\operatorname{arccosh}(x)) \right)}$$