

MIT Integration Bee: Finals
(Time limit per integral: 5 minutes)

Finals Problem 1

$$\lim_{n \rightarrow \infty} \int_0^1 \left(\sum_{k=1}^n \frac{1}{nx^2 + kx + n} \right) dx$$

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$$\lim_{n \rightarrow \infty} \int_0^1 \left(\sum_{k=1}^n \frac{1}{nx^2 + kx + n} \right) dx = \boxed{\frac{5\pi^2}{72}}$$

Finals Problem 2

$$\int \frac{dx}{(x-1)\sqrt[4]{x^3+x}}$$

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$$\int \frac{dx}{(x-1)\sqrt[4]{x^3+x}}$$
$$= \frac{1}{4\sqrt[4]{2}} \left(\log \left(\frac{x+1-\sqrt[4]{8x^3+8x}}{x+1+\sqrt[4]{8x^3+8x}} \right) + 2 \arctan \left(\frac{\sqrt[4]{8x^3+8x}}{x+1} \right) \right)$$

Finals Problem 3

$$\lim_{n \rightarrow \infty} \int_0^{2026} \underbrace{\log_{\sqrt{2}}(x + \log_{\sqrt{2}}(x + \cdots \log_{\sqrt{2}}(x + 2026) \cdots))}_{n \text{ logs}} dx$$

Finals Problem 3

$$\lim_{n \rightarrow \infty} \int_0^{2026} \underbrace{\log_{\sqrt{2}}(x + \log_{\sqrt{2}}(x + \cdots \log_{\sqrt{2}}(x + 2026) \cdots))}_{n \text{ logs}} dx$$

$$= \boxed{44806 - \frac{4088}{\log(2)}}$$

Finals Problem 4

$$\int_0^{1/4} \left(\left[\sqrt[4]{\frac{1}{x} - 4} \right]^2 + \left[\sqrt[4]{\frac{1}{x} - 4} \right] \right) dx$$

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$$\int_0^{1/4} \left(\left[\sqrt[4]{\frac{1}{x} - 4} \right]^2 + \left[\sqrt[4]{\frac{1}{x} - 4} \right] \right) dx = \boxed{\frac{3}{4}}$$

Finals Problem 5

$$\int_{-\infty}^{\infty} \left(\left(\frac{1}{x-2} + \frac{3}{x-4} + \frac{5}{x-6} \right)^{-2} + 1 \right)^{-1} dx$$

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$$\int_{-\infty}^{\infty} \left(\left(\frac{1}{x-2} + \frac{3}{x-4} + \frac{5}{x-6} \right)^{-2} + 1 \right)^{-1} dx = \boxed{9\pi}$$