MIT Integration Bee: Regular Season
(Time limit per integral: 2 minutes)
Regular Season Problem 1

\[ \int_1^{2024} \left\lfloor \log_{43}(x) \right\rfloor \, dx \]
Regular Season Problem 1

\[ \int_{1}^{2024} \left\lfloor \log_{43}(x) \right\rfloor \, dx = 2156 \]
Regular Season Problem 2

\[ \int \frac{dx}{x^{2024} - x^{4047}} \]
Regular Season Problem 2

\[
\int \frac{dx}{x^{2024} - x^{4047}} = \log(x) - \frac{1}{2023} \log(1 - x^{2023}) - \frac{1}{2023}x^{-2023}
\]
Regular Season Problem 3

\[ \int_{0}^{1} x^2 (1 - x)^{2024} \, dx \]
Regular Season Problem 3

\[ \int_0^1 x^2 (1 - x)^{2024} \, dx = \frac{2}{2027 \cdot 2026 \cdot 2025} \]
Regular Season Problem 4

\[
\int \frac{2023x + 1}{x^2 + 2024} \, dx
\]
Regular Season Problem 4

\[ \int \frac{2023x + 1}{x^2 + 2024} \, dx = \frac{2023}{2} \log(x^2 + 2024) + \frac{1}{\sqrt{2024}} \arctan\left(\frac{x}{\sqrt{2024}}\right) \]
Regular Season Problem 5

\[ \int_0^{\pi/2} \sec^2(x) e^{-\sec^2(x)} \, dx \]
Regular Season Problem 5

\[
\int_0^{\pi/2} \sec^2(x)e^{-\sec^2(x)} \, dx = \frac{\sqrt{\pi}}{2e}
\]
Regular Season Problem 6

\[ \int \cot(x) \cot(2x) \, dx \]
Regular Season Problem 6

\[
\int \cot(x) \cot(2x) \, dx = -x - \frac{\cot(x)}{2}
\]
Regular Season Problem 7

\[ \int \frac{\sinh^2(x)}{\tanh(2x)} \, dx \]
Regular Season Problem 7

\[
\int \frac{\sinh^2(x)}{\tanh(2x)} \, dx = \frac{1}{4} \cosh(2x) - \frac{1}{2} \log(\cosh(x))
\]
Regular Season Problem 8

\[ \int \arctan(\sqrt{x}) \, dx \]
Regular Season Problem 8

\[ \int \arctan(\sqrt{x}) \, dx = (x + 1) \arctan(\sqrt{x}) - \sqrt{x} \]
Regular Season Problem 9

$$\int_0^\infty \frac{x \log(x)}{x^4 + 1} \, dx$$
Regular Season Problem 9

\[
\int_0^\infty \frac{x \log(x)}{x^4 + 1} \, dx = 0
\]
Regular Season Problem 10

\[ \int_{0}^{10} \lfloor x \lfloor x \rfloor \rfloor \, dx \]
Regular Season Problem 10

\[ \int_{0}^{10} \lfloor x \rfloor \, dx = 303 \]
Regular Season Problem 11

\[ \int_{0}^{1} e^{-x} \sqrt{1 + \cot^2(\arccos(e^{-x}))} \, dx \]
Regular Season Problem 11

\[
\int_0^1 e^{-x} \sqrt{1 + \cot^2(\arccos(e^{-x}))} \, dx = \frac{\pi}{2} - \arcsin(e^{-1})
\]
Regular Season Problem 12

\[
\int_{1}^{3} \frac{1 + \frac{1 + \cdots}{x + \cdots}}{x + \frac{1 + \cdots}{x + \cdots}} \, dx
\]
Regular Season Problem 12

\[ \int_{1}^{3} \frac{1 + \frac{1 + \cdots}{x + \cdots}}{x + \frac{1 + \cdots}{x + \cdots}} \, dx = \sqrt{2} - 1 + \log(1 + \sqrt{2}) \]
Regular Season Problem 13

\[ \int_0^1 \frac{2x(1-x)^2}{1+x^2} \, dx \]
Regular Season Problem 13

\[
\int_0^1 \frac{2x(1-x)^2}{1+x^2} \, dx = \pi - 3
\]
Regular Season Problem 14

\[ \int e^{ex} + 3x \; dx \]
Regular Season Problem 14

\[ \int e^{e^x + 3x} \, dx = (e^{2x} - 2e^x + 2)e^x \]
Regular Season Problem 15

\[ \int_{-\sqrt{3}/2}^{\sqrt{3}/2} 2 \left( 1 - \frac{|x|}{\sqrt{3}} \right) \, dx \]
Regular Season Problem 15

\[ \int_{-\sqrt{3}/2}^{\sqrt{3}/2} 2 \left( 1 - \frac{|x|}{\sqrt{3}} \right) \, dx = \frac{3\sqrt{3}}{2} \]
Regular Season Problem 16

\[
\int \frac{\log(1 + x^2)}{x^2} \, dx
\]
Regular Season Problem 16

\[ \int \frac{\log(1 + x^2)}{x^2} \, dx = 2 \arctan(x) - \frac{\log(1 + x^2)}{x} \]
Regular Season Problem 17

\[ \int 2^x x^2 \, dx \]
Regular Season Problem 17

\[ \int 2^x x^2 \, dx = \frac{2^x}{\log^3(2)} \left( x^2 \log^2(2) - 2x \log(2) + 2 \right) \]
Regular Season Problem 18

\[ \int_{0}^{1} \sqrt{x^8 - x^6 + x^4} \cdot \sqrt{1 + x^2} \, dx \]
Regular Season Problem 18

\[ \int_0^1 \sqrt{x^8 - x^6 + x^4} \cdot \sqrt{1 + x^2} \, dx \]

\[ = \frac{1}{6} \left( \sqrt{2} + \log(1 + \sqrt{2}) \right) \]
Regular Season Problem 19

\[ \int_{1}^{\infty} \frac{e^{x} + xe^{x}}{x^{2}e^{2x} - 1} \, dx \]
Regular Season Problem 19

\[ \int_{1}^{\infty} \frac{e^x + xe^x}{x^2e^{2x} - 1} \, dx = \frac{1}{2} \log \left( \frac{e + 1}{e - 1} \right) \]
Regular Season Problem 20

\[ \int_{0}^{\infty} (80x^3 - 60x^4 + 14x^5 - x^6)e^{-x} \, dx \]
Regular Season Problem 20

\[ \int_{0}^{\infty} (80x^3 - 60x^4 + 14x^5 - x^6)e^{-x} \, dx = 0 \]