MIT Integration Bee: Regular Season

(Time limit per integral: 2 minutes)

Regular Season Problem 1

$$\int_1^{2024} \lfloor \log_{43}(x) \rfloor \, dx$$

Regular Season Problem 1

$$\int_1^{2024} \lfloor \log_{43}(x) \rfloor \, dx = \boxed{2156}$$

Regular Season Problem 2

$$\int \frac{dx}{x^{2024} - x^{4047}}$$

Regular Season Problem 2

$$\int \frac{dx}{x^{2024} - x^{4047}}$$

$$= \boxed{\log(x) - \frac{1}{2023}\log(1 - x^{2023}) - \frac{1}{2023}x^{-2023}}$$

Regular Season Problem 3

$$\int_0^1 x^2(1-x)^{2024}\,dx$$

Regular Season Problem 3

$$\int_0^1 x^2(1-x)^{2024}\,dx = \boxed{\frac{2}{2027 \cdot 2026 \cdot 2025}}$$

Regular Season Problem 4

$$\int \frac{2023x + 1}{x^2 + 2024}\,dx$$

Regular Season Problem 4

$$\int \frac{2023x+1}{x^2+2024}\,dx$$

$$= \boxed{\frac{2023}{2}\log(x^2+2024) + \frac{1}{\sqrt{2024}}\arctan\left(\frac{x}{\sqrt{2024}}\right)}$$

Regular Season Problem 5

$$\int_0^{\pi/2} \sec^2(x) e^{-\sec^2(x)}\, dx$$

Regular Season Problem 5

$$\int_0^{\pi/2} \sec^2(x) e^{-\sec^2(x)}\, dx = \boxed{\frac{\sqrt{\pi}}{2e}}$$

Regular Season Problem 6

$$\int \cot(x)\cot(2x)\,dx$$

Regular Season Problem 6

$$\int \cot(x)\cot(2x)\,dx = \boxed{-x - \frac{\cot(x)}{2}}$$

Regular Season Problem 7

$$\int \frac{\sinh^2(x)}{\tanh(2x)}\, dx$$

Regular Season Problem 7

$$\int \frac{\sinh^2(x)}{\tanh(2x)}\,dx = \boxed{\frac{1}{4}\cosh(2x) - \frac{1}{2}\log(\cosh(x))}$$

Regular Season Problem 8

$$\int \arctan(\sqrt{x})\,dx$$

Regular Season Problem 8

$$\int \arctan(\sqrt{x})\,dx = \boxed{(x+1)\arctan(\sqrt{x}) - \sqrt{x}}$$

Regular Season Problem 9

$$\int_0^\infty \frac{x \log(x)}{x^4+1}\, dx$$

Regular Season Problem 9

$$\int_0^\infty \frac{x\log(x)}{x^4+1}\,dx = \boxed{0}$$

Regular Season Problem 10

$$\int_0^{10} \lfloor x \lfloor x \rfloor \rfloor \, dx$$

Regular Season Problem 10

$$\int_0^{10} \lfloor x \lfloor x \rfloor \rfloor \, dx = \boxed{303}$$

Regular Season Problem 11

$$\int_0^1 e^{-x}\sqrt{1+\cot^2(\arccos(e^{-x}))}\,dx$$

Regular Season Problem 11

$$\int_0^1 e^{-x}\sqrt{1+\cot^2(\arccos(e^{-x}))}\,dx = \boxed{\frac{\pi}{2} - \arcsin(e^{-1})}$$

Regular Season Problem 12

$$\int_1^3 \frac{1+\frac{1+\cdots}{x+\cdots}}{x+\frac{1+\cdots}{x+\cdots}}\,dx$$

Regular Season Problem 12

$$\int_1^3 \frac{1+\frac{1+\cdots}{x+\cdots}}{x+\frac{1+\cdots}{x+\cdots}}\,dx = \boxed{\sqrt{2}-1+\log(1+\sqrt{2})}$$

Regular Season Problem 13

$$\int_0^1 \frac{2x(1-x)^2}{1+x^2}\,dx$$

Regular Season Problem 13

$$\int_0^1 \frac{2x(1-x)^2}{1+x^2}\,dx = \boxed{\pi - 3}$$

Regular Season Problem 14

$$\int e^{e^x+3x}\,dx$$

Regular Season Problem 14

$$\int e^{e^x+3x}\,dx = \boxed{(e^{2x} - 2e^x + 2)e^{e^x}}$$

Regular Season Problem 15

$$\int_{-\sqrt{3}/2}^{\sqrt{3}/2} 2\left(1 - \frac{|x|}{\sqrt{3}}\right)\, dx$$

Regular Season Problem 15

$$\int_{-\sqrt{3}/2}^{\sqrt{3}/2} 2\left(1-\frac{|x|}{\sqrt{3}}\right)\,dx = \boxed{\frac{3\sqrt{3}}{2}}$$

Regular Season Problem 16

$$\int \frac{\log(1+x^2)}{x^2}\,dx$$

Regular Season Problem 16

$$\int \frac{\log(1+x^2)}{x^2}\,dx = \boxed{2\arctan(x) - \frac{\log(1+x^2)}{x}}$$

Regular Season Problem 17

$$\int 2^x x^2 \, dx$$

Regular Season Problem 17

$$\int 2^x x^2 \, dx = \boxed{\frac{2^x}{\log^3(2)}\left(x^2 \log^2(2) - 2x \log(2) + 2\right)}$$

Regular Season Problem 18

$$\int_0^1 \sqrt{x^8 - x^6 + x^4} \cdot \sqrt{1 + x^2}\, dx$$

Regular Season Problem 18

$$\int_0^1 \sqrt{x^8 - x^6 + x^4} \cdot \sqrt{1 + x^2}\, dx$$

$$= \boxed{\frac{1}{6}\left(\sqrt{2} + \log(1 + \sqrt{2})\right)}$$

Regular Season Problem 19

$$\int_{1}^{\infty} \frac{e^{x} + xe^{x}}{x^{2}e^{2x} - 1}\,dx$$

Regular Season Problem 19

$$\int_1^\infty \frac{e^x + xe^x}{x^2e^{2x} - 1}\,dx = \boxed{\frac{1}{2}\log\left(\frac{e+1}{e-1}\right)}$$

Regular Season Problem 20

$$\int_0^\infty (80x^3 - 60x^4 + 14x^5 - x^6)e^{-x}\,dx$$

Regular Season Problem 20

$$\int_0^\infty (80x^3 - 60x^4 + 14x^5 - x^6)e^{-x}\,dx = \boxed{0}$$