MIT Integration Bee: Quarterfinal #1

(Time limit per integral: 3 minutes)

Quarterfinal #1 Problem 1

$$\int \log(x)\left(\left(\frac{x}{e}\right)^x + \left(\frac{e}{x}\right)^x\right)\,dx$$

Quarterfinal #1 Problem 1

$$\int \log(x) \left(\left(\frac{x}{e}\right)^x + \left(\frac{e}{x}\right)^x \right) \, dx = \boxed{\left(\frac{x}{e}\right)^x - \left(\frac{e}{x}\right)^x}$$

Quarterfinal #1 Problem 2

$$\int_0^\infty \frac{\sin^3(x)}{x}\,dx$$

Quarterfinal #1 Problem 2

$$\int_0^\infty \frac{\sin^3(x)}{x}\,dx = \boxed{\frac{\pi}{4}}$$

Quarterfinal #1 Problem 3

$$\int \begin{vmatrix} x & 1 & 0 & 0 & 0 \\ 1 & x & 1 & 0 & 0 \\ 0 & 1 & x & 1 & 0 \\ 0 & 0 & 1 & x & 1 \\ 0 & 0 & 0 & 1 & x \end{vmatrix} \, dx$$

Quarterfinal #1 Problem 3

$$\int \begin{vmatrix} x & 1 & 0 & 0 & 0 \\ 1 & x & 1 & 0 & 0 \\ 0 & 1 & x & 1 & 0 \\ 0 & 0 & 1 & x & 1 \\ 0 & 0 & 0 & 1 & x \end{vmatrix} \, dx = \boxed{\frac{x^6}{6} - x^4 + \frac{3x^2}{2}}$$

MIT Integration Bee: Quarterfinal Tiebreakers

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Quarterfinal Tiebreakers Problem 1

$$\int_0^{2024} x^{2024} \log_{2024}(x)\, dx$$

Quarterfinal Tiebreakers Problem 1

$$\int_0^{2024} x^{2024} \log_{2024}(x)\, dx = \boxed{\frac{2024^{2025}}{2025} - \frac{2024^{2025}}{2025^2 \log(2024)}}$$

Quarterfinal Tiebreakers Problem 2

$$\lim_{t\to\infty}\int_0^2 \left(x^{-2024t}\prod_{n=1}^{2024}\sin(nx^t)\right)\,dx$$

Quarterfinal Tiebreakers Problem 2

$$\lim_{t\to\infty} \int_0^2 \left(x^{-2024t} \prod_{n=1}^{2024} \sin(nx^t) \right) dx = \boxed{2024!}$$

MIT Integration Bee: Quarterfinal #2

(Time limit per integral: 3 minutes)

Quarterfinal #2 Problem 1

$$\lim_{n\to\infty}\left(\int_0^1 \sum_{k=1}^{n} \frac{(kx)^4}{n^5}\,dx\right)$$

Quarterfinal #2 Problem 1

$$\lim_{n\to\infty}\left(\int_0^1 \sum_{k=1}^{n} \frac{(kx)^4}{n^5}\,dx\right) = \boxed{\frac{1}{25}}$$

Quarterfinal #2 Problem 2

$$\int_0^1 \frac{\log(1+x^2+x^3+x^4+x^5+x^6+x^7+x^9)}{x}\,dx$$

Quarterfinal #2 Problem 2

$$\int_0^1 \frac{\log(1+x^2+x^3+x^4+x^5+x^6+x^7+x^9)}{x}\,dx$$

$$= \boxed{\frac{13\pi^2}{144}}$$

Quarterfinal #2 Problem 3

$$\int_0^1 \left(1 - \sqrt[2024]{x}\right)^{2024} dx$$

Quarterfinal #2 Problem 3

$$\int_0^1 \left(1 - \sqrt[2024]{x}\right)^{2024} dx = \boxed{\frac{1}{\binom{4048}{2024}}}$$

MIT Integration Bee: Quarterfinal #3

(Time limit per integral: 3 minutes)

Quarterfinal #3 Problem 1

$$\int_0^{2\pi} \left| \left\{ \lfloor \sin x \rfloor, \lfloor \cos x \rfloor, \lfloor \tan x \rfloor, \lfloor \cot x \rfloor \right\} \right| dx$$

Quarterfinal #3 Problem 1

$$\int_0^{2\pi} \left| \left\{ \lfloor \sin x \rfloor, \lfloor \cos x \rfloor, \lfloor \tan x \rfloor, \lfloor \cot x \rfloor \right\} \right| dx = \boxed{\frac{11}{2}\pi}$$

Quarterfinal #3 Problem 2

$$\int_0^\infty \frac{dx}{(x+1)\left(\log^2(x)+\pi^2\right)}$$

Quarterfinal #3 Problem 2

$$\int_0^\infty \frac{dx}{(x+1)\left(\log^2(x)+\pi^2\right)} = \boxed{\frac{1}{2}}$$

Quarterfinal #3 Problem 3

$$\lim_{n\to\infty} \frac{1}{n} \int_0^n \max\left(\{x\}, \left\{\sqrt{2}x\right\}, \left\{\sqrt{3}x\right\}\right) dx$$

Quarterfinal #3 Problem 3

$$\lim_{n\to\infty} \frac{1}{n} \int_0^n \max\left(\{x\}, \left\{\sqrt{2}x\right\}, \left\{\sqrt{3}x\right\}\right)\, dx = \boxed{\frac{3}{4}}$$

MIT Integration Bee: Quarterfinal #4

(Time limit per integral: 3 minutes)

Quarterfinal #4 Problem 1

$$\int \frac{e^{2x}}{(1-e^{x})^{2024}}\,dx$$

Quarterfinal #4 Problem 1

$$\int \frac{e^{2x}}{(1-e^x)^{2024}}\,dx$$

$$= \boxed{\frac{1}{2023(1-e^x)^{2023}} - \frac{1}{2022(1-e^x)^{2022}}}$$

Quarterfinal #4 Problem 2

$$\lim_{n\to\infty} \log_n \left(\int_0^1 (1 - x^3)^n \, dx \right)$$

Quarterfinal #4 Problem 2

$$\lim_{n\to\infty} \log_n \left(\int_0^1 (1 - x^3)^n \, dx \right) = \boxed{-\frac{1}{3}}$$

Quarterfinal #4 Problem 3

$$\int \frac{\sin x}{1+\sin x} \cdot \frac{\cos x}{1+\cos x}\,dx$$

Quarterfinal #4 Problem 3

$$\int \frac{\sin x}{1+\sin x} \cdot \frac{\cos x}{1+\cos x}\, dx = \boxed{x + \ln\left(\frac{1+\cos x}{1+\sin x}\right)}$$