MIT Integration Bee: Finals

(Time limit per integral: 5 minutes)

Finals Problem 1

$$\int \frac{e^{x/2}\cos x}{\sqrt[3]{3\cos x + 4\sin x}}\,dx$$

Finals Problem 1

$$\int \frac{e^{x/2}\cos x}{\sqrt[3]{3\cos x + 4\sin x}}\,dx = \boxed{\frac{6}{25}(3\cos x + 4\sin x)^{2/3}e^{x/2}}$$

Finals Problem 2

$$\int_0^\infty \frac{\log(2e^x - 1)}{e^x - 1}\, dx$$

Finals Problem 2

$$\int_0^\infty \frac{\log(2e^x - 1)}{e^x - 1}\,dx = \boxed{\frac{\pi^2}{4}}$$

Finals Problem 3

$$\int_{-\infty}^{\infty} \frac{dx}{x^4 + x^3 + x^2 + x + 1}$$

Finals Problem 3

$$\int_{-\infty}^{\infty} \frac{dx}{x^4 + x^3 + x^2 + x + 1} = \boxed{\frac{\sqrt{10 + 2\sqrt{5}}}{5}\pi}$$

Finals Problem 4

$$\int_{-1/3}^{1} \left(\sqrt[3]{1+\sqrt{1-x^3}} + \sqrt[3]{1-\sqrt{1-x^3}} \right) \, dx$$

Finals Problem 4

$$\int_{-1/3}^{1} \left(\sqrt[3]{1+\sqrt{1-x^3}} + \sqrt[3]{1-\sqrt{1-x^3}} \right) \, dx$$

$$= \boxed{\frac{14}{9} + \frac{2}{3} \log 2}$$

Finals Problem 5

$$\int_0^1 \max_{n\in\mathbb{Z}_{\geq 0}} \left(\frac{1}{2^n} \left(\lfloor 2^n x \rfloor - \left\lfloor 2^n x - \frac{1}{4} \right\rfloor \right) \right) dx$$

Finals Problem 5

$$\int_0^1 \max_{n\in\mathbb{Z}_{\geq 0}} \left(\frac{1}{2^n}\left(\lfloor 2^n x\rfloor - \left\lfloor 2^n x - \frac{1}{4}\right\rfloor\right)\right)\, dx = \boxed{\frac{4}{11}}$$

MIT Integration Bee: Finals Tiebreakers

(Time limit per integral: 5 minutes)

Finals Tiebreakers Problem 1

$$\int \frac{dx}{\sqrt[4]{x^4+1}}$$

Finals Tiebreakers Problem 1

$$\int \frac{dx}{\sqrt[4]{x^4+1}}$$

$$= \boxed{\frac{1}{2}\arctan\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{4}\log\left(\frac{\sqrt[4]{1+x^4}+x}{\sqrt[4]{1+x^4}-x}\right)}$$

Finals Tiebreakers Problem 2

$$\int_0^{2\pi} \frac{(\sin 2x - 5\sin x)\sin x}{\cos 2x - 10\cos x + 13}\,dx$$

Finals Tiebreakers Problem 2

$$\int_0^{2\pi} \frac{(\sin 2x - 5\sin x)\sin x}{\cos 2x - 10\cos x + 13}\, dx = \boxed{(2\sqrt{2} + \sqrt{3} - 5)\pi}$$

Finals Tiebreakers Problem 3

$$\int \sqrt{x^4 - 4x + 3}\, dx$$

Finals Tiebreakers Problem 3

$$\int \sqrt{x^4 - 4x + 3}\, dx$$

$$= \boxed{\tfrac{1}{3}(x^2+2x+3)^{3/2}-(x+1)\sqrt{x^2+2x+3}-2\log(\sqrt{x^2+2x+3}+x+1)}$$

Finals Tiebreakers Problem 4

$$\int_{-\infty}^{\infty} \sin^2(2^x)\cos^2(3^x)\left(4\cos^2(2^x)(4\cos^2(3^x)-3)^2-1\right)dx$$

Finals Tiebreakers Problem 4

$$\int_{-\infty}^{\infty} \sin^2(2^x)\cos^2(3^x)\left(4\cos^2(2^x)(4\cos^2(3^x)-3)^2-1\right)dx = \boxed{\frac{1}{4}}$$

Finals Tiebreakers Problem 5

$$\int_2^\infty \frac{\lfloor x \rfloor x^2}{x^6 - 1}\,dx$$

Finals Tiebreakers Problem 5

$$\int_2^\infty \frac{\lfloor x \rfloor x^2}{x^6 - 1}\, dx = \boxed{\frac{1}{6} \log \frac{27}{14}}$$

MIT Integration Bee: Lightning Round

(Time limit per integral: 1 minute)

Lightning Round Problem 1

$$\int_0^1 \left(\left(1 - x^{\frac{3}{2}}\right)^{\frac{3}{2}} - \left(1 - x^{\frac{2}{3}}\right)^{\frac{2}{3}} \right) dx$$

Lightning Round Problem 1

$$\int_0^1 \left(\left(1 - x^{\frac{3}{2}}\right)^{\frac{3}{2}} - \left(1 - x^{\frac{2}{3}}\right)^{\frac{2}{3}} \right) dx = \boxed{0}$$

Lightning Round Problem 2

$$\int \left(\frac{x}{x-1} \right)^4 \, dx$$

Lightning Round Problem 2

$$\int \left(\frac{x}{x-1}\right)^4 dx$$

$$= \boxed{x + 4\log(x-1) - \frac{6}{x-1} - \frac{2}{(x-1)^2} - \frac{1}{3(x-1)^3}}$$

Lightning Round Problem 3

$$\int \frac{(\tan(1012x)+\tan(1013x))\cos(1012x)\cos(1013x)}{\cos(2025x)}\,dx$$

Lightning Round Problem 3

$$\int \frac{(\tan(1012x)+\tan(1013x))\cos(1012x)\cos(1013x)}{\cos(2025x)}\,dx$$

$$= \boxed{-\frac{\log(\cos(2025x))}{2025}}$$