

MIT Integration Bee: Quarterfinal #1
(Time limit per integral: 2 minutes)

Quarterfinal #1 Problem 1

$$\int_0^1 \frac{x^4(1-x)^2}{1+x^2} dx$$

Quarterfinal #1 Problem 1

$$\int_0^1 \frac{x^4(1-x)^2}{1+x^2} dx = \boxed{\frac{7}{10} - \log 2}$$

Quarterfinal #1 Problem 2

$$\int \left(\begin{array}{cccc} \cos(3x) & \cos(5x) & \cos(6x) & \cos(7x) \\ -\cos(x) & \cos(2x) & \cos(4x) & \cos(8x) \end{array} \right) dx$$

Quarterfinal #1 Problem 2

$$\int \left(\begin{array}{l} \cos(3x) \cos(5x) \cos(6x) \cos(7x) \\ - \cos(x) \cos(2x) \cos(4x) \cos(8x) \end{array} \right) dx$$
$$= \frac{1}{8} \left(\frac{\sin(21x)}{21} - \frac{\sin(13x)}{13} \right)$$

MIT Integration Bee: Quarterfinal #2
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Quarterfinal #2 Problem 1

$$\int_{\sqrt{e}}^{\infty} x^{-\log x} dx$$

Quarterfinal #2 Problem 1

$$\int_{\sqrt{e}}^{\infty} x^{-\log x} dx = \boxed{\frac{\sqrt[4]{e\pi^2}}{2}}$$

Quarterfinal #2 Problem 2

$$\int \frac{1 - 2x}{(1 + x)^2 x^{2/3}} dx$$

Quarterfinal #2 Problem 2

$$\int \frac{1 - 2x}{(1 + x)^2 x^{2/3}} dx = \boxed{\frac{3x^{1/3}}{1 + x}}$$

Quarterfinal #2 Problem 3

$$\lim_{n \rightarrow \infty} \left(\frac{1}{n} \int_0^n \cos^2 \left(\frac{\pi x^2}{\sqrt{2}} \right) dx \right)$$

Quarterfinal #2 Problem 3

$$\lim_{n \rightarrow \infty} \left(\frac{1}{n} \int_0^n \cos^2 \left(\frac{\pi x^2}{\sqrt{2}} \right) dx \right) = \boxed{\frac{1}{2}}$$

MIT Integration Bee: Quarterfinal #3
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Quarterfinal #3 Problem 1

$$\int_0^{2^{10}} \sum_{n=0}^{\infty} \left\{ \frac{x}{2^n} \right\} dx$$

Quarterfinal #3 Problem 1

$$\int_0^{2^{10}} \sum_{n=0}^{\infty} \left\{ \frac{x}{2^n} \right\} dx = \boxed{12 \cdot 2^9 = 6144}$$

Quarterfinal #3 Problem 2

$$\int_0^{\infty} \operatorname{sech}^2(x + \tan(x)) \, dx$$

Quarterfinal #3 Problem 2

$$\int_0^{\infty} \operatorname{sech}^2(x + \tan(x)) \, dx = \boxed{1}$$

MIT Integration Bee: Quarterfinal #4
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Quarterfinal #4 Problem 1

$$\int_0^{\pi/2} \frac{dx}{1 + \cos x + \sin x}$$

Quarterfinal #4 Problem 1

$$\int_0^{\pi/2} \frac{dx}{1 + \cos x + \sin x} = \boxed{\log 2}$$

Quarterfinal #4 Problem 2

$$\lim_{\varepsilon \rightarrow 0^+} \left(\varepsilon^4 \int_0^{\pi/2 - \varepsilon} \tan^5(x) dx \right)$$

Quarterfinal #4 Problem 2

$$\lim_{\varepsilon \rightarrow 0^+} \left(\varepsilon^4 \int_0^{\pi/2 - \varepsilon} \tan^5(x) dx \right) = \boxed{\frac{1}{4}}$$

Quarterfinal #4 Problem 3

$$\int_0^1 \left[\sqrt{1 + \frac{1}{x}} \right] dx$$

Quarterfinal #4 Problem 3

$$\int_0^1 \left[\sqrt{1 + \frac{1}{x}} \right] dx = \boxed{\frac{7}{4}}$$