MIT Integration Bee: Regular Season

(Time limit per integral: 2 minutes)

Regular Season Problem 1

$$\int_0^{100} \left\lceil \sqrt{x} \right\rceil \, dx$$

Regular Season Problem 1

$$\int_0^{100} \left\lceil \sqrt{x} \right\rceil \, dx = \boxed{715}$$

Regular Season Problem 2

$$\int \frac{\log(1+x)}{x^2}\,dx$$

Regular Season Problem 2

$$\int \frac{\log(1+x)}{x^2}\,dx = \boxed{\log(x) - \frac{(x+1)\log(x+1)}{x}}$$

Regular Season Problem 3

$$\int_{\frac{\pi}{2}-1}^{\frac{\pi}{2}+1} \cos(\arcsin(\arccos(\sin(x))))\,dx$$

Regular Season Problem 3

$$\int_{\frac{\pi}{2}-1}^{\frac{\pi}{2}+1} \cos(\arcsin(\arccos(\sin(x))))\,dx = \boxed{\frac{\pi}{2}}$$

Regular Season Problem 4

$$\int_{-2}^{2} |(x-2)(x-1)x(x+1)(x+2)|\, dx$$

Regular Season Problem 4

$$\int_{-2}^{2} |(x-2)(x-1)x(x+1)(x+2)|\, dx = \boxed{\frac{19}{3}}$$

Regular Season Problem 5

$$\int 2020\sin^{2019}(x)\cos^{2019}(x)-8084\sin^{2021}(x)\cos^{2021}(x)\,dx$$

Regular Season Problem 5

$$\int 2020 \sin^{2019}(x) \cos^{2019}(x) - 8084 \sin^{2021}(x) \cos^{2021}(x)\, dx$$

$$= \boxed{\sin^{2020}(x) \cos^{2022}(x) - \sin^{2022}(x) \cos^{2020}(x)}$$

Regular Season Problem 6

$$\int \frac{3x^3 + 2x^2 + 1}{\sqrt[3]{x^3 + 1}}\, dx$$

Regular Season Problem 6

$$\int \frac{3x^3+2x^2+1}{\sqrt[3]{x^3+1}}\,dx = \boxed{(x+1)(x^3+1)^{\frac{2}{3}}}$$

Regular Season Problem 7

$$\int \frac{1}{\sin^4 x \cos^4 x}\, dx$$

Regular Season Problem 7

$$\int \frac{1}{\sin^4 x \cos^4 x}\, dx = \boxed{-8 \cot 2x - \frac{8}{3} \cot^3 2x}$$

Regular Season Problem 8

$$\int \frac{x + \sin x}{1 + \cos x}\, dx$$

Regular Season Problem 8

$$\int \frac{x + \sin x}{1 + \cos x}\, dx = \boxed{x \tan \frac{x}{2}}$$

Regular Season Problem 9

$$\int \sinh^3 x \cosh^2 x \, dx$$

Regular Season Problem 9

$$\int \sinh^3 x \cosh^2 x \, dx = \boxed{\frac{1}{5}(\sinh^2 x - \frac{2}{3}) \cosh^3 x}$$

Regular Season Problem 10

$$\int 4^x 3^{2^x} \, dx$$

Regular Season Problem 10

$$\int 4^x 3^{2^x}\,dx = \boxed{\frac{3^{2^x}}{\log 2}\left(\frac{2^x}{\log 3} - \frac{1}{(\log 3)^2}\right)}$$

Regular Season Problem 11

$$\int \frac{\cos(x) - \sin(x)}{2 + \sin(2x)}\, dx$$

Regular Season Problem 11

$$\int \frac{\cos(x) - \sin(x)}{2 + \sin(2x)}\, dx = \boxed{\arctan(\sin(x) + \cos(x))}$$

Regular Season Problem 12

$$\int \frac{\sec^2(1+\log x) - \tan(1+\log x)}{x^2}\,dx$$

Regular Season Problem 12

$$\int \frac{\sec^2(1+\log x) - \tan(1+\log x)}{x^2}\,dx = \boxed{\frac{\tan(1+\log x)}{x}}$$

Regular Season Problem 13

$$\int_0^1 \sqrt{\frac{1}{x}\log\frac{1}{x}}dx$$

Regular Season Problem 13

$$\int_0^1 \sqrt{\frac{1}{x}\log\frac{1}{x}}dx = \boxed{\sqrt{2\pi}}$$

Regular Season Problem 14

$$\sum_{n=2}^{\infty} \int_0^{\infty} \frac{(x-1)x^n}{1+x^n+x^{n+1}+x^{2n+1}}\, dx$$

Regular Season Problem 14

$$\sum_{n=2}^{\infty} \int_0^{\infty} \frac{(x-1)x^n}{1+x^n+x^{n+1}+x^{2n+1}}\,dx = \boxed{\frac{\pi}{2}-1}$$

Regular Season Problem 15

$$\int_0^{2\pi} (1 - \cos(x))^5 \cos(5x) \, dx$$

Regular Season Problem 15

$$\int_0^{2\pi} (1-\cos(x))^5 \cos(5x)\,dx = \boxed{-\frac{\pi}{16}}$$

Regular Season Problem 16

$$\int_0^{10} \lceil x \rceil \left(\max_{k \in \mathbb{Z}_{\geq 0}} \frac{x^k}{k!} \right) dx$$

Regular Season Problem 16

$$\int_0^{10} \lceil x \rceil \left(\max_{k \in \mathbb{Z}_{\geq 0}} \frac{x^k}{k!} \right) dx = \boxed{\frac{10^{10}}{9!}}$$

Regular Season Problem 17

$$\int \frac{4\sin x + 3\cos x}{3\sin x + 4\cos x}\,dx$$

Regular Season Problem 17

$$\int \frac{4\sin x + 3\cos x}{3\sin x + 4\cos x}\,dx = \boxed{\frac{24x - 7\log(3\sin x + 4\cos x)}{25}}$$

Regular Season Problem 18

$$\int_{-1}^{1} \sqrt{4 - (1 + |x|)^2} - (\sqrt{3} - \sqrt{4 - x^2})\, dx$$

Regular Season Problem 18

$$\int_{-1}^{1} \sqrt{4-(1+|x|)^2} - (\sqrt{3} - \sqrt{4-x^2})\, dx = \boxed{2\pi - 2\sqrt{3}}$$

Regular Season Problem 19

$$\int x^2 \sin(\log x)\, dx$$

Regular Season Problem 19

$$\int x^2 \sin(\log x)\,dx = \boxed{\frac{x^3}{10}(3\sin(\log x) - \cos(\log x))}$$

Regular Season Problem 20

$$\int_0^\infty (36x^5 - 12x^6 + x^7)e^{-x}\,dx$$

Regular Season Problem 20

$$\int_0^\infty (36x^5 - 12x^6 + x^7)e^{-x}\,dx = \boxed{720}$$