MIT Integration Bee: Quarterfinal #1

(Time limit per integral: 2 minutes)

Quarterfinal #1 Problem 1

$$\int_1^{2022} \frac{\{x\}}{x}\, dx$$

Quarterfinal #1 Problem 1

$$\int_1^{2022} \frac{\{x\}}{x}\,dx = \boxed{2021 - \log\left(\frac{2022^{2021}}{2021!}\right)}$$

Quarterfinal #1 Problem 2

$$\lim_{n \to \infty} n \int_0^{\frac{\pi}{4}} \tan^n(x)\, dx$$

Quarterfinal #1 Problem 2

$$\lim_{n\to\infty} n \int_0^{\frac{\pi}{4}} \tan^n(x)\, dx = \boxed{\frac{1}{2}}$$

Quarterfinal #1 Problem 3

$$\int_0^\infty \frac{x^{1010}}{(1+x)^{2022}}\,dx$$

Quarterfinal #1 Problem 3

$$\int_0^\infty \frac{x^{1010}}{(1+x)^{2022}}\,dx = \boxed{\frac{1010!^2}{2021!}}$$

MIT Integration Bee: Quarterfinal #2

(Time limit per integral: 2 minutes)

Quarterfinal #2 Problem 1

$$\int \arcsin(x) \arccos(x)\, dx$$

Quarterfinal #2 Problem 1

$$\int \arcsin(x)\arccos(x)\,dx$$

$$= \boxed{x(\arcsin(x)\arccos(x)+2)+\sqrt{1-x^2}(\arccos(x)-\arcsin(x))}$$

Quarterfinal #2 Problem 2

$$\max_{\{x_1,x_2,x_3,x_4,x_5,x_6,x_7\}=\{1,2,3,4,5,6,7\}} \int_{\int_{x_1}^{x_2} x_3\,dx}^{\int_{x_4}^{x_5} x_6\,dx} x_7\,dx$$

Quarterfinal #2 Problem 2

$$\max_{\{x_1,x_2,x_3,x_4,x_5,x_6,x_7\}=\{1,2,3,4,5,6,7\}} \int_{\int_{x_1}^{x_2} x_3\,dx}^{\int_{x_4}^{x_5} x_6\,dx} x_7\,dx = \boxed{217}$$

Quarterfinal #2 Problem 3

$$\lim_{n\to\infty} \sqrt[n]{\int_0^2 (1+6x-7x^2+4x^3-x^4)^n \, dx}$$

Quarterfinal #2 Problem 3

$$\lim_{n\to\infty} \sqrt[n]{\int_0^2 (1+6x-7x^2+4x^3-x^4)^n \, dx} = \boxed{3}$$

MIT Integration Bee: Quarterfinal #3

(Time limit per integral: 2 minutes)

Quarterfinal #3 Problem 1

$$\int_0^1 \frac{x^4}{\sqrt{1-x}}\,dx$$

Quarterfinal #3 Problem 1

$$\int_0^1 \frac{x^4}{\sqrt{1-x}}\,dx = \boxed{\frac{256}{315}}$$

Quarterfinal #3 Problem 2

$$\int_0^{\infty} \left\lceil \frac{1}{\lceil x \rceil - x} \right\rceil^{-\lceil x \rceil} dx$$

Quarterfinal #3 Problem 2

$$\int_0^\infty \left\lceil \frac{1}{\lceil x \rceil - x} \right\rceil^{-\lceil x \rceil} dx = \boxed{\frac{\pi^2}{6} - 1}$$

Quarterfinal #3 Problem 3

$$\lim_{n\to\infty} n \int_0^{\infty} \sin\left(\frac{1}{x^n}\right)\, dx$$

Quarterfinal #3 Problem 3

$$\lim_{n\to\infty} n\int_0^\infty \sin\left(\frac{1}{x^n}\right)\,dx = \boxed{\frac{\pi}{2}}$$

MIT Integration Bee: Quarterfinal #4

(Time limit per integral: 2 minutes)

Quarterfinal #4 Problem 1

$$\int_0^1 \frac{-x + \sqrt{4 - 3x^2}}{2} \, dx$$

Quarterfinal #4 Problem 1

$$\int_0^1 \frac{-x + \sqrt{4 - 3x^2}}{2}\, dx = \boxed{\frac{\pi}{3\sqrt{3}}}$$

Quarterfinal #4 Problem 2

$$\lim_{n\to\infty} \sqrt{n} \int_{-\frac{1}{2}}^{\frac{1}{2}} (1 - 3x^2 + x^4)^n \, dx$$

Quarterfinal #4 Problem 2

$$\lim_{n\to\infty} \sqrt{n} \int_{-\frac{1}{2}}^{\frac{1}{2}} (1 - 3x^2 + x^4)^n \, dx = \boxed{\sqrt{\frac{\pi}{3}}}$$

Quarterfinal #4 Problem 3

$$\int_{\frac{1}{2022}}^{2022} \frac{1+x^2}{x^2+x^{2022}}\,dx$$

Quarterfinal #4 Problem 3

$$\int_{\frac{1}{2022}}^{2022} \frac{1+x^2}{x^2+x^{2022}}\,dx = \boxed{2022 - \frac{1}{2022}}$$