Project Summary

Intellectual Merit

The proposed project is centered about two seemingly different subfields of low dimensional topology; contact topology and Heegaard Floer homologies. Both subfields have been inspired by physics albeit in rather different ways. Contact manifolds arise quite naturally as the phase space of a moving object. While the origins of Heegaard Floer homological invariants are deeper, they are obtained by associating to an object a configuration space of constrained functions and computing the spaces algebro-topological invariants. Not only low dimensional topology is inspired by physics but it describes our physical world; the space we live in is a 3–dimensional manifold while space-time is a 4–dimensional manifold. Thus low dimensional topology has been long studied, and it has proven to be hard. It is interestingly in many aspects more complicated than higher dimensional topology.

The relationship between contact topology and various Floer homologies has been a fundamental tool to settle open questions in low-dimensional topology and contact topology. The contact invariant in the Heegaard Floer homology groups was one of the main instruments for these applications. The proposed project is devoted to deepen this connection, by understanding further relations of the two subjects. The hoped results will have consequences not only in contact geometry and Heegaard Floer homology, but in more basic low dimensional topology too. In particular the P. I. would like to prove, that if a rational homology sphere contains an incompressible surface, then it is not an L-space. She tries to develop a “contact” version of Morse theory, that allows one to give a completely geometrical descriptions of contact structures on a surface times the interval. Using these results she can classify Legendrian representations of a wide class of knots. Also she wants to prove the equivalence of the two possible generalizations of overtwistedness in higher dimensions.

Broader Impacts

The Principal Investigator (P.I.). has always been involved in Math Circles, and started to led them in 1997. At least five of her former students remained in Mathematics, and started their PhD. studies recently. The P.I. is still in contact and informally helps one of them; Béla Rácz, who just started his graduate studies in topology at Princeton University. The P.I. plans to get involved in a Math Circle activities in the Boston area too. She has already been asked and accepted to give a talk for ”Euclid Lab”, a new Internet based research opportunity for high and middle school students. In addition the P.I. has already written two expository articles in a Hungarian journal. She plans to write another expository article on contact geometry. The P.I. not only works to advertise Mathematics for general audience, she gave several talks on seminars at various universities. Some of the talks were specifically aimed at graduate students. She also gave a mini course on Heegaard Floer homologies at the University of Cape town. She organized a seminars at each place she spent time at since she graduated. The P.I. leds problem sessions on the Summer Graduate Workshop on Symplectic and Contact Geometry and Topology at MSRI in 2009. She has been asked to led Problem Session at the Trimester on Contact and Symplectic Topology in 2011 in Nantes.

Project Description

Introduction

My proposed research concentrates on two subfields of low dimensional topology: Heegaard Floer homology and contact geometry. Here I give a brief description of both topics.

Heegaard Floer homology is a package of 3– and 4–dimensional invariants originally defined by Ozsváth and Szabó [34, 33, 36] and later modified and generalized to knots by Ozsváth and Szabó [32, 37], and Rasmussen [41], to 3–manifolds with decorated boundary by Juhász [21], to 3–manifolds with parametrized boundary by Lipshitz, Ozsváth and Thurston[40, 39]. The most general framework called bordered sutured Floer homology including most of the above is being developed by Zarev [46].

Contact geometry enables 3–manifolds with an extra structure that allows finer classification question for both 3–manifolds and knots in 3-manifolds. Knots that respect the contact structure are called Legendrian knots. Contact geometry first arose in physics through the work of Sophus Lie in the nineteens century, and recently started to flourish by the introduction of new tools, such as open books and convex surfaces [16].

Although these two subfields are quite different both in their origins and theories they have been proven to be useful when one applied to the other. On one hand there are several invariants in Heegaard Floer theory defined for contact 3–manifolds that reflects topological properties of a contact structure or a Legendrian knot in a contact 3-manifold. These invariants allowed us to distinguish contact structures, classify contact structures on a given 3–manifold and give examples of 3–manifolds that have no contact structures of a certain type. The invariants of Legendrian knots in Heegaard Floer homology gave several first proofs of new phenomenons in knot theory. During this project I would like to understand relations between the already defined knot invariants for Legendrian knots (see Subsection 4), find vanishing criteria for these invariants (see Subsection 5)) see how these invariants are related for concordant Legendrian knots (see Subsection 6) and use these invariants to distinguish and hopefully (with other tools) classify the Legendrian representations of a wide class of knots (see Subsection 3). On the other hand contact geometry provided a tool for proving results in Heegaard Floer homology. The fact that Heegaard Floer homology determines the Seifert genus of a knot was first proved with the help of contact 3–manifolds (Ozsváth-Szabó [31]). Later Ghiggini [14] and Ni [29] proved using techniques from contact geometry to prove that Heegaard Floer homology detects fibered knots. As part of the proposed projects together with bordered Floer homology I would like to use similar tools as the above three papers to give a lover bound for the rank of Heegaard Floer homology for manifold that contain an incompressible surface (see Subsection 2). Of course both theories are interesting in their own rights, so I propose research projects including only each one of them. Using bordered Floer homology I would like to understand the growth rate of the ranks of the Heegaard Floer homology groups for the sequence of 3–manifolds that are obtained by iterating a gluing map along a surface (see Subsection 1). On the contact geometry side my project is in the direction of getting a better understanding of contact structures on higher dimensional manifolds (see Subsection 7, and Subsection 8).

Background and Previous Research

Heegaard Floer theories

Heegaard Floer homologies, (Ozsváth-Szabó, [33, 34, 36]) the recently-discovered invariants for 3– and 4–manifolds, come from an application of Lagrangian Floer homology to spaces associated to Heegaard diagrams. The boundary map of the resulting homology theory therefore relies on a delicate count of (almost) holomorphic surfaces in (almost) complex manifolds. Although this theory is conjecturally isomorphic to Seiberg-Witten theory, it is more topological and combinatorial in its flavor and thus easier to work with in certain contexts. These homologies admit generalizations and refinements for knots (Ozsváth-Szabó [32] and Rasmussen [41]) and links (Ozsváth-Szabó [37]) in 3–manifolds and for non-closed 3–manifolds with certain boundary conditions (Juhász [21]), called sutured Floer homology. Last year a gluable version of Heegaard Floer homology was developed by Lipshitz, Ozsváth and Thurston [40, 39]. Bordered Floer homology has a parametrized boundary, and allows one to keep track of all the holomorphic surfaces that meet the boundary. This makes the theory a bit clumsy, and although it has already proven to be useful in theory [40, 46], it is hard to carry out concrete computations.

The 4–dimensional version of the Heegaard Floer homology package [36] gives maps between Heegaard Floer homologies of cobordant 3–manifolds. For sutured 3-manifolds [22] the cobordism are cornered. The boundary is divided into 3 parts; the two original 3-manifolds, and a cobordism of the boundaries of the 3-manifolds. In order to define the map between the sutured Heegaard Floer homologies one needs to introduce a contact structure ξ on the part of the boundary that is the cobordism of the boundaries of the 3-manifolds. Depending on this decoration one gets different mappings Φ_{Z_ξ}.

Contact 3–manifolds

Although contact geometry was born in the late 19th century in the work of Sophus Lie, it has just recently started to develop rapidly, with the discovery of convex surface theory and by recognizing its role in other parts of topology. For example Property P for knots —a possible first step for resolving the Poincaré conjecture— was proved using contact 3–manifolds (Kronheimer-Mrowka [24]). Also, the fact that Heegaard Floer homology determines the Seifert genus of a knot was first proved with the help of contact 3–manifolds (Ozsváth-Szabó [31]). Being the natural boundaries of Stein domains, the use of contact 3–manifolds resulted in a topological description of Stein-manifolds [6]. A *contact structure* ξ on an oriented 3–manifold Y is a totally non-integrable plane field. In other words it is a plane distribution that is not everywhere tangent to any open embedded surface. Contact structures can be locally given as the kernel of a 1–form α. The totally non integrability condition then translates to $\alpha \wedge d\alpha > 0$. In the following we will assume, that α can be given globally. Such contact structures are called *cooriented contact structures*. In dimension 3 we distinguish two types of contact structures, tight contact structures and overtwisted contact structures. A contact structure is called *overtwisted* if it contains a disc whose tangent plane field along its boundary coincides with ξ (See Figure 1). Any 3–manifold admits a contact structure (Martinet [26]), but the obvious construction usually gives overtwisted contact structures. Overtwisted contact structures satisfy an h-principle [5], thus they can be classified using only homological data. It is more subtle though to understand the set of all different tight contact structures on a given 3–manifold. One way to understand them is by examining lower dimension submanifolds that respect the structure

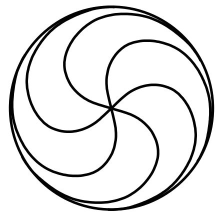

Figure 1: Overtwisted disc

in a way. Every surface embedded in a contact 3–manifold inherits a singular foliation from ξ; the intersection of the tangent planes of the surface with the contact planes . This foliation is called the *characteristic foliation* of the surface (See Figure 1 for the characteristic foliation of an overtwisted disc), and it determines the contact structure in a neighborhood of the surface. There are surfaces, along which an even easier structure, a properly embedded 1–manifold the *dividing curve* is enough to describe the contact structure in their neighborhood (Giroux [16]). Such surfaces are called *convex surfaces*. These are surfaces with an I-invariant contact structure in their neighborhood. The dividing curve is given by those points where ξ contains the I-direction. Thus convex surfaces became the right boundary conditions for contact 3–manifolds. The elementary change of the dividing curves while moving the convex surface is called *bypass attachment*. Roughly speaking a bypass is the attachment of the neighborhood of a half overtwisted disc (see Figure 2) to the convex boundary of a contact structure. Bypasses do not change the topological type of a manifold, but can change the contact structure. Bypasses were used to classify contact structures [17], and Legendrian knots [12, 11]. A good way to describe contact structures is through *open book decompositions*. An

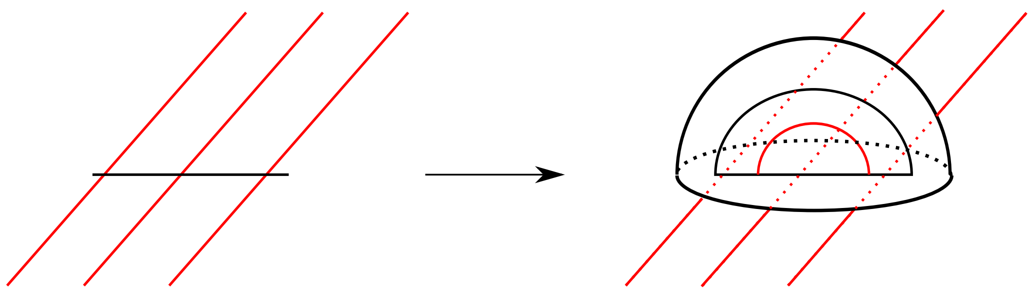

Figure 2: Bypass attachment

open book decomposition of a 3–manifold, is a fibration of a link in the manifold. Thus an open book decomposition can be given as a surface S (the *page* of the open book) with boundary (this is the link or also called the *binding* of the open book) and a monodromy map $h\colon S \to S$ that fixes a neighborhood of ∂S. A contact structure ξ is *compatible* with an open book decomposition, if the link is transverse to ξ and α is positive on the pages of the open book. There is a strong relation between contact structures and open book decompositions [16] of a 3–manifold. Moreover many properties of contact structures, such as Stein-fillability and overtwistedness can be read off from its open book decompositions.

In Heegaard Floer homology contact invariants were defined for contact 3–manifolds without (Ozsváth-Szabó [35]) or with (Honda-Kazez-Matic [18]) boundary. These invariants had many applications for example a recent one is a new proof for the fact that a contact 3–manifold having

Giroux torsion cannot be Stein-fillable (Ghiggini-Honda-Van Horn-Morris [15]).

Contact structures can be defined on any odd dimensional manifold as totally non-integrable hyperplane distributions. Higher dimensional contact structures also have open book decompositions [20], but they are way harder to understand in general. It is not known which manifolds admit contact structures, and most theorems and notions that are true in dimension three fail to generalize to higher dimensions.

Legendrian and transverse knots

There are two ways for a one dimensional submanifold to respects the contact structure. Its tangents can entirely lie in the plane distribution, in which case the knot is called *Legendrian knot*, or if the tangents are transverse to the planes, the knot is then called a *transverse knot*. A Legendrian knot with a given knot type has two classical invariants: its Thurston-Bennequin number and its rotation number. While for transverse knots there is only one invariant; the self-linking number. The problem of classifying Legendrian (transverse) knots up to Legendrian (transverse) isotopy naturally leads to the question whether these invariants classify Legendrian (transverse) knots. A knot type is called *Legendrian (transverse) simple* if any two realizations of it with equal classical invariants are Legendrian (transverse) isotopic. The unknot (Eliashberg-Fraser [7]), torus knots and the figure-eight knot (Etnyre-Honda [12]) were proved to be both Legendrian and transversely simple. By constructing a new invariant for Legendrian knots, Chekanov [3] showed that not all knots are Legendrian simple, in particular he proved that the knot $m(5_2)$ is not Legendrian simple. Later many other Legendrian non-simple knots were found (Epstein-Fuchs-Meyer [9] and Ng [27]). In [10] we give a complete classification of Legendrian representations of all twist knots, these are the first infinite class of non Legendrian simple knots with a complete classification. Among other results we prove:

Theorem 1 (Etnyre-Ng-Vértesi [10]) *The $m((2n+1)_2)$ knot has exactly $\lceil \frac{n^2}{2} \rceil$ different Legendrian representations with maximal Thurston-Bennequin number.*

The case for transverse knots turned out to be harder. Birman and Menasco [1], and Etnyre and Honda [11] constructed families of transversely non-simple knots using braid and convex surface theory. The Legendrian invariant in the combinatorial Floer homology provided another tool to construct transversely non-simple knots (Ng-Ozsváth-Thurston [28]) By proving a connected sum formula for the combinatorial Legendrian invariant, I proved, the existence of infinitely many transversely non-simple knots:

Theorem 2 (Vértesi [45]) *There exist infinitely many transversely non-simple knots.*

The definition of the contact invariant in Heegaard Floer homology admits a generalization for Legendrian and transverse knots $\widehat{\mathcal{L}}$ in the knot Floer homology (Lisca-Ozsváth-Stipsicz-Szabó [25]). The contact invariant of Honda, Kazez and Matic for the complement of a Legendrian knot gives rise to a Legendrian invariant: the EH-class. With Stipsicz we understood the relation between these two invariants:

Theorem 3 (Stipsicz-Vértesi [43]) *There is a map from the sutured Floer homology for the knot-complement to the knot Floer homology mapping $\widehat{\mathcal{L}}$ to* EH.

A nice consequence of this theorem, which was independently obtained by Vela-Vick [44], is the following:

Theorem 4 (Stipsicz-Vértesi [43]) *If the knot complement contains Giroux torsion, then* $\widehat{\mathcal{L}}$ *vanishes.*

Research Problems and Proposed Approaches

My proposed research problems are all related to the two subfields Heegaard Floer homology and contact geometry.

1 The growth rate of the rank of Heegaard Floer homology

A recent development in Heegaard Floer homology [40, 39] allows one to glue two 3–manifolds with parametrized boundary and understand the Heegaard Floer homology of the glued up manifold. I would like to understand this gluing in one of the easiest settings when one cuts a 3–manifold to two pieces along a surface and re glues the pieces using the iterations of a map $\phi\colon F \to F$. (See Figure 3.) I am interested in the growth rate of the rank of the Heegaard Floer homology in terms

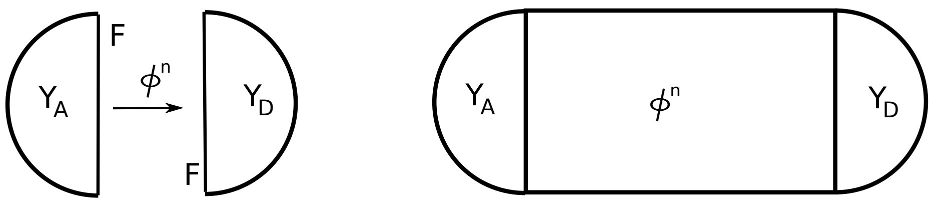

Figure 3: Y^n : Obtaining a new 3–manifold by reglueing with ϕ^n

of n. Namely:

Conjecture 5 *For the 3–manifolds* Y_n *of Figure 3 the rank of Heegaard Floer homology* $rk(HF(Y_n))$ *grows exponentially in* n*.*

This problem is a warm up question for the next problem in Subsection 2.

Approach

Given a 3–manifold Y, and a separating surface F in it. A parametrization of F gives rise to an $\mathcal{A}_\infty$–algebra $\mathcal{A}(F)$. The two pieces Y_A and Y_D have boundaries parametrized by a surface F and $-F$, thus as in [40] one can associate an $\mathcal{A}_\infty$ module $\widehat{CFA}_{\mathcal{A}(F)}(Y_A)$, and type D module ${}^{\mathcal{A}(-F)}\widehat{CFD}(Y_D)$ to the pieces Y_A and Y_D. The homology of the derived tensor product $H_*\left(\widehat{CFA}_{\mathcal{A}(F)}(Y_A)\widetilde{\otimes}\,{}^{\mathcal{A}(-F)}\widehat{CFD}(Y_D)\right)$ gives back $HF(Y_A \cup_F Y_D)$. Now the map ϕ defines a parametrization of the boundary of $F\times I$, and thus a DA bimodule ${}^{\mathcal{A}(-F)}\widehat{CFDA}(\phi)_{\mathcal{A}(F)}$. One model for this bimodule is given in [39]. The Heegaard Floer homology of the manifold $Y_n = Y_A \cup_{\phi^n} Y_D$ is the homology of the chain complex:

$$\widehat{CFA}_{\mathcal{A}(F)}(Y_A)\ \widetilde{\otimes}\ \left({}^{\mathcal{A}(-F)}\widehat{CFDA}(\phi)_{\mathcal{A}(F)}\right)^{\widetilde{\otimes}^n}\ \widetilde{\otimes}\ {}^{\mathcal{A}(-F)}\widehat{CFD}(Y_D)$$

I hope to get a good approximation of the rank of the $\widehat{HF}$ only using information on the AD bimodule

$$\left({}^{\mathcal{A}(-F)}\widehat{CFDA}(\phi)_{\mathcal{A}(F)}\right)^{\widetilde{\otimes}^n}.$$

This approximation depends on a good encoding of the arising DA bimodule. For instance if the Heegaard diagram for $F \times I$ is nice (in the sense of [42]) the DA bimodule is an honest differential module over a differential graded algebra, and the tensor product becomes an honest tensor product of bimodules. As of now I can compute the above when the cutting surface F is a torus, and understand the (already known) growth rate for lens spaces with the above described method.

2 The rank of HF for 3–manifolds containing an incompressible surface

If a 3–manifold contains an incompressible surface, then one expects it to be complicated, for instance to have high rank Heegaard Floer homology groups. Indeed, using knot Floer homology Eftekhary [4] proved that if a prime homology sphere contains an incompressible torus then the Heegaard Floer homology has rank at least two. With Ghiggini we would like to get this result for 3–manifolds with incompressible surfaces of arbitrary genus.

Conjecture 6 *Let Y be a homology sphere. If Y contains a separating incompressible surface then $rk(HF(Y)) \geq 2$*

In particular:

Conjecture 7 *An L-space contains no incompressible surface.*

This result would be one of the first geometrical applications of bordered Floer homology.

Approach

Most results concerning the rank of Heegaard Floer homology groups use contact structures. Ozsváth-Szabó [31] proved that knot Floer homology determines the Seifert genus of a knot. Namely the genus of the knot is the highest grading where knot Floer homology does not vanish. The non vanishing is proved by showing a contact structure whose contact invariant is in the grading given by the genus of the knot. This proof was reinvented in a harder settings when Ghiggini [14] and Ni [29] proved that knot Floer homology detects fibered knots. They proved a conjecture of Ozsváth and Szabó that states that a knot is fibered if and only if its knot Floer homology is rank 1 in the grading given by the genus of the knot (i.e. $rk(HFK(K, g)) = 1$). The main tool for the proof, besides knot Floer homology and the contact invariant, was Gabai's sutured manifold theory [13]. Roughly speaking a sutured manifold (Y, Γ) is a 3-manifold together with a collection Γ of disjoint curves on ∂Y. If F is a properly embedded surface in Y (which intersects the boundary in a good way) then F induces a new suture Γ' on $Y' = Y \setminus F$. Gabai proved that if (Y, Γ) is taut, then after a finite number of cuts it can be decomposed into a bunch of balls with a unique curve on the boundary. Moreover, from that decomposition we can construct a taut foliation on Y such that $Y \setminus F$ is union of leaves, and F is transverse to the foliation. The idea of the proof of [14] and [29] was to construct two essentially different sutured manifold decompositions on the complement of a non fibered knot (or more precisely on the 3–manifold obtained by zero surgery on the knot). Then perturb the two resulting taut foliations into symplectically fillable contact structures using the theorem of Eliashberg and Thurston [8]. Finally show that the two contact invariants give linearly independent elements in $HFK(K, g)$. The two sutured manifold decompositions started both with a genus minimizing Seifert surface of K, but differentiated at the second step. In fact the second cut was performed along suitable modifications of the same properly embedded surface with positive genus taken with a different orientation in each decomposition.

In our settings however there is no natural suture Γ given on F, so sutured manifold in its own cannot deal with the problem. I hope to be able to choose the sutures in a way, so that the sutured manifold is taut. As the surface is incompressible the none of the halves is going to be fibered, thus SFH will have more than one generators that are contact invariants of the modified taut foliations. According to Zarev [46] bordered Floer homology unifies these sutured theories. Thus one can glue the resulting contact invariants together. I would like to show that, out of the several tensors of contact invariants more than one will not vanish.

3 Legendrian classification of some knot types

The techniques developed in [10] can be used to classify Legendrian representatives of (non-closed) braids. The words defined by the Legendrian braids form a submonoid of the framed braid group. I want to give a simple set of generators and relations for this monoid. This classification should also be useful in the classifications of some closed braids. Among other results with Etnyre we can classify transverse representatives of any braiding along a transversely simple uniformly thick knot. Although this result sounds technical there is evidence that we can do more and classify Legendrian representations of many knot types, such as positive braids, or braids with few strands.

Approach

In the standard contact structure on $D^2 \times I$ Legendrian isotopy classes of Legendrian braids are in one-to-one correspondence with isotopy classes of contact structures. Thus the classification of Legendrian braids can be done, by using contact geometrical methods, in particular convex surfaces and bypasses. We were able to prove that Legendrian representations of braids are built up from 3 types of building block depicted on Figure 4. Thus Legendrian representations of braids can be

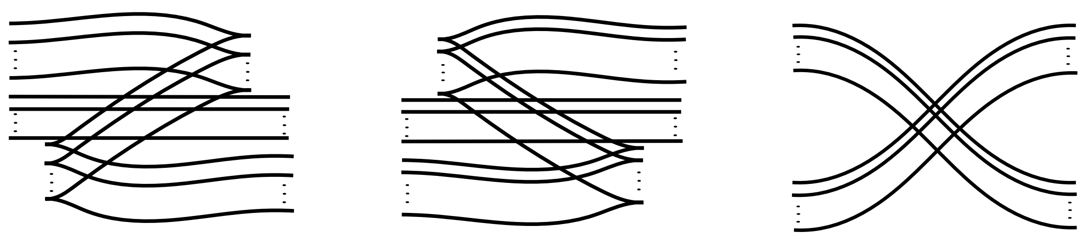

Figure 4: Building blocks of Legendrian braids (all other strands are horizontal)

encoded in a word of these building blocks. A Legendrian (transverse) braid is naturally framed, thus the words defined by the Legendrian (transverse) braids form a submonoid of the framed braid group. The existence of bypasses and relations between bypasses can be used to define generators and relations of the Legendrian monoid for few strands.

In order to understand Legendrian braids along a knot one needs to cut a neighborhood of the knot along a disc to obtain a standard $D^2 \times I$. There is a technical condition which makes this cut and neighborhood automatically exist. A solid torus $D^2 \times S^1$ is said to represent a knot if its core is isotopic to the knot. Consider all solid tori with convex boundary representing a given knot type. Then there is a common trivialization of their boundary with the meridional and the Seifert framing, thus one can look at the slopes $s(\Gamma)$ of the dividing curves on the boundary. The *width* of a topological knot type K is given as

$$w(K) = \sup_{D^2\times S^1 \text{ represents } K} \frac{1}{s(\Gamma)}$$

A knot K is called *uniformly thick* if the maximal Thurston-Bennequin number equals $w(K)$ and if all representations of K can be thickened up to a solid torus that has dividing curve of slope $w(K)$. Unfortunately the unknot is not uniformly thick, thus the cutting disc is not automatic to find. One aim of further research is to find the correct neighborhood and cutting disc for positive braids along the unknot too. Once the braid is in the standard $D^2 \times I$ the result of [10] says that positive braids are transverse simple. Thus one only needs to see, that the self linking number of the braided knot and the self linking number of the original knot and the braid pattern determine each other. The exact same thing was proved for torus knots in [12] and the proof will directly generalize to our setting. So we will be able to prove:

Conjecture 8 *Positive braids of a uniformly thick Legendrian simple knot are transversely simple.*

This result is one of similar types, when one sees that simpleness is inherited by certain topological operations on knots. The main aim of the whole project is something we can only obtain if we figure out how to deal with cables of the unknot. We would like to prove a conjecture of Kálmán [23] which says:

Conjecture 9 *Positive braids are Legendrian simple.*

4 Relations between Legendrian invariants

Using the language of Heegaard Floer homology recently three different invariants were defined for Legendrian and transverse knots. One in the combinatorial settings of knot Floer homology for the 3–sphere [38]: $\widehat{\lambda}$, one in knot Floer homology for a general contact 3–manifold [25]: $\widehat{\mathcal{L}}$ and one defined as the contact invariant associated to the knot-complement: EH. With András Stipsicz [43] we understood the connection between the last two of them; there is a map between the homologies sending EH to $\widehat{\mathcal{L}}$. This suggests, that EH contains more information about a Legendrian knot than $\widehat{\mathcal{L}}$. Morally EH includes all surgery information of the knot. However there is no known examples, that can be distinguished by the EH-class but not by $\widehat{\mathcal{L}}$. The 5_2 knot seems to be a good candidate for proving the difference of these invariants. In the standard contact 3–sphere the first two invariants, though behave fairly similarly; both of them is in the knot Floer homology, for an unoriented knot there are naturally two of each, they vanish under the same kind of operations, etc. Thus it is conjectured that:

Conjecture 10 *For a Legendrian knot L in the 3–sphere $\widehat{\lambda}(L) = \widehat{\mathcal{L}}(K)$.*

The result of [44] would immediately translate to:

Conjecture 11 $\widehat{\lambda} \neq 0$ *for any Legendrian approximation of a binding component of an open book for the standard contact structure.*

The previous conjecture could give obstructions for specific transverse knots to be bindings of open books, since $\widehat{\lambda}$ is easy to compute, and its vanishing is even easier to check. In general, this result can be a good tool to construct counterexamples since $\widehat{\lambda}$ is easy to compute, while $\widehat{\mathcal{L}}$ can be used to prove geometrical statements.

Approach

This project is to be carried out in a collaboration with Stipsicz. Both invariants have a concrete description; $\widehat{\lambda}$ is defined through grid diagrams on the torus, while $\widehat{\mathcal{L}}$ is described using an open book of the standard contact structure, with the knot being on its page. The proof of the equality should go by finding an open book that is related to the toroidal grid diagram by a "nice" sequence of Heegaard moves, under which the transformation of the invariant can be tracked.

5 Vanishing of the Legendrian invariant

In an ongoing research with Baldwin and Etnyre we define Heegaard Floer invariants for arcs, and assign elements of this homology for Legendrian representations of arcs. Generalizing the methods of [15], we hope to get various vanishing and even non-vanishing results for the Legendrian invariant [25].

Approach

There is nothing to stop one from defining a sutured version of Heegaard Floer knot homology, which is then a homology theory of properly embedded arcs in a sutured 3-manifold. The analog of the Legendrian invariant $\widehat{\mathcal{L}}$ in this settings gives an invariant of Legendrian arcs. If we have a knot or arc glued up from two pieces sitting in two sutured 3–manifolds then the map of [19] or [22] generalizes to this setting giving a map from the tensor of the two arc Floer homologies to the arc (or knot) Floer homology of the glued up object. This map can be even better understood by the work of Zarev [46]. It can be computed that the defined Legendrian arc invariant vanishes for some amazingly simple (open) Legendrian braids. This automatically proves the vanishing of $\widehat{\mathcal{L}}$ for a wide class of knots. By understanding the map given by the embedding better, we hope to get non vanishing results in some special cases. The construction of Ng, Ozsváth and Thurston [28] for non Legendrian simple knots had Legendrian knots that differed only at a small piece. Understanding the Legendrian arc invariant for those pieces would not only give another proof for non simplicity, but –by inserting the pieces into different knots– would provide a new infinite class of non Legendrian simple (prime) knots.

6 Lagrangian concordance

The generalization of concordance for Legendrian knots has been recently started to be studied [2]. For a given contact 3–manifold (Y, ξ) let L_0 and L_1 be Legendrian knots, we say that L_0 is Lagrangian concordant to L_1 if there exists a Lagrangian cylinder $F : S^1 \times \mathbb{R} \hookrightarrow Y$ satisfying:

$$\exists T > 0 \text{ such that } \; F|_{(-\infty,-T)} = L_0 \times (-\infty, -T) \; \textit{and} \; F|_{(T,\infty)} = L_1 \times (T, \infty).$$

(See figure 5.) Since we work in the symplectization, Lagrangian concordance is not symmetric. An example for this phenomenon was provided by Chantraine [2]. Thus unlike its smooth version

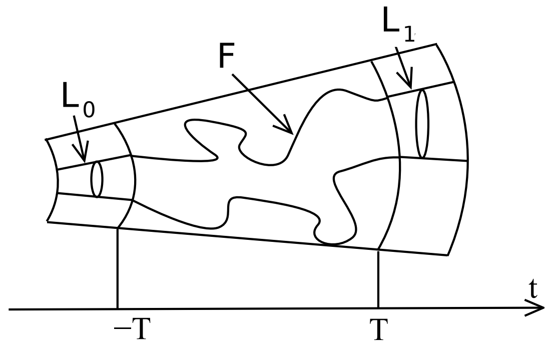

Figure 5: Lagrangian Concordance in $(Y \times I, \omega = d(e^t\alpha))$.

Lagrangian concordance does not give an equivalence relation [2], rather a partial ordering of Legendrian knots. The classical invariant are preserved under concordance. With Levine we would like to understand the behavior of the Legendrian invariant $\widehat{\mathcal{L}}$ [25] under concordance:

Conjecture 12 *Suppose L_0 is Lagrangian concordant to L_1. Then one can define a contact structure on $Z = \partial N(F)$ such that the map Φ_{ξ_Z} defined by Juhász on $\overline{W} = \overline{Y \times [-N, N] \setminus N(F)}$ brings $\widehat{\mathcal{L}}(L_1)$ to $\widehat{\mathcal{L}}(L_0)$.*

This relation would give a new proof for the fact that concordance is not an equivalence relation.

Approach

The construction of this map is a particular case of the map defined by Juhász [22] between sutured Floer homologies of contact 3–manifolds which is defined as follows. A cobordism of sutured 3–manifolds (Y_0, Γ_0) and (Y_1, Γ_1) is a 4–manifold W with boundary and corners. The boundary ∂W is separated to 3 pieces $-Y_0 \cup Z \cup Y_1$ where Z is a cobordism between ∂Y_0 and ∂Y_1. The map between sutured Floer homologies is well defined after a choice of compatible contact structure ξ_Z on Z. This means that $\partial Z = -\partial Y_0 \cup \partial Y_1$ is convex with respect to ξ_Z and the dividing curves on ∂Y_0 and ∂Y_1 are respectively Γ_0 and Γ_1. Juhász's map $\Phi_{\xi_Z}\colon SFH(Y_0, \Gamma_0) \to SFH(Y_1, \Gamma_1)$ is a common generalization of the cobordism map of ordinary Heegaard Floer homology and the map defined by Honda Kazez and Matic [19]. For our purposes a *Weinstein cobordism* between sutured manifolds is a sutured cobordism between $-Y_0 \cup -Z$ and Y_1, that is built up from Weinstein one– and two–handles along a Legendrian link L with framing tb(L)-1. (See [22] for a complete definition.)

The boundary Z of the neighborhood of F is naturally a contact manifold (Z, ξ). Thus $Y \times \mathbb{R} \setminus N(F)$ is a sutured cobordism connecting the complement of the standard neighborhood of L_0: $Y \setminus N(L_0)$ to the standard neighborhood of L_1: $Y \setminus N(L_1)$. In this case the dividing curves Γ_0 and Γ_1 on $\partial Y \setminus N(L_0)$ and on $Y \setminus N(L_1)$ are given by the Thurston-Bennequin framing. Thus the contact structure defines a map

$$\Phi_{Z_\xi} : SFH(Y \setminus N(L_0), \Gamma_0) \to SFH(Y \setminus N(L_1), \Gamma_1).$$

Thus if we can endow this sutured cobordism with a Weinstein structure it is clear, that with the inherited contact structure ξ on Z maps $EH(L_0)$ to $EH(L_1)$. I would like to use this map to see that the $\widehat{\mathcal{L}}$ are also mapped to each other (with a different contact structure on Z):

Conjecture 13 *One can define a contact structure on Z such that $\Phi_{\xi_Z}(\widehat{\mathcal{L}}(L_1)) = \widehat{\mathcal{L}}(L_0)$.*

Note that since that $\widehat{\mathcal{L}}$ lives in the sutured Floer homology associated to complement of the knot with meridional sutures, thus there is no natural contact structure on Z.

7 Bypass attachments defining the same contact structure

The current proof (which is not published) that all contact structures on a surface $F \times I$ are built up from bypasses, uses ODE-methods. By developing contact Morse-theory, I would like to give a topological proof that bypasses are indeed the elementary building blocks of contact structures. This treatment could lead to a new definition of the contact TQFT defined in [19] and its generalization [22]. I would also like to relate sets of bypasses defining a the same contact structure. As above any contact structure ξ on $F \times I$ can be obtained as a sequence of bypass attachments. Intuitively bypasses are the contact analogs of handles in Morse theory. Similarly to Morse theory there are changes of this sequences which does not change the isotopy class of the contact structure. One of them is an attachment of a trivial bypass anywhere in the sequence. The other move is commutation, which is that whenever there are two consecutive bypasses in the sequence such that the upper one is not attached on the top of the lower one we can change their order. It is believed that these two moves form a complete set of moves:

Conjecture 14 *Suppose that two series of bypass attachments define the same contact structure on Y. Then the series of bypass attachments are related by commutation of bypasses and trivial bypass attachments.*

Note that the bypass rotation move is a trivial bypass attachment after some commutation.

Approach

Bypasses can be nicely interpreted in terms of contact Morse functions. A Morse function f on a contact manifold is called *contact* [16], if it admits a gradient like flow X that preserves the contact structure. The level sets of contact Morse functions are convex hypersurfaces. And the critical points lie on a codimension 2 submanifold of the space. This submanifold is called the *critical manifold* and it consists of points such that $X \in \xi$. Intuitively the critical submanifold is the union of the dividing surfaces of all level sets. Any two smooth Morse(-Smale) function f_0 and f_1 can be connected by a path of functions f_t ($t \in [0,1]$) such that f_t is Morse(-Smale) for all but finitely many values of t. In the non Morse(-Smale) points we have a complete understanding on the change the Morse(-Smale) function experiences. There is either a path between two critical points of the same index. This results in a handle slide. Or there can be a path between two critical points with consecutive indices. This results in handle cancellation. I would like to develop a similar Morse theoretical picture for contact Morse functions:

Conjecture 15 *If two contact Morse functions f_0, f_1 with gradient like vector fields X_0 and X_1 define the same contact 3–manifold (Y, ξ). Then there is a paths f_t ($t \in [0,1]$) of functions and X_t ($t \in [0,1]$) contact vector fields. Such that in all but finitely many points f_t is Morse and X_t is gradient like for f_t.*

There are several elementary proofs for the fact that there is an almost always Morse path connecting two Morse functions. I try to use those to find a good way of deforming X_t along with f_t. So far I am only able to find X_t for paths which leave the set of critical points unchanged. If I can understand the above two kinds of degenerations locally in the contact case I should be able to patch the contact vector fields together. A bypass in the contact Morse theoretical settings can be interpreted as a canceling pair of a contact one and two handle. On $F \times I$ the critical points must cancel each other in pairs, thus a contact Morse function defines a sequence of bypass attachments. And for any contact cell decomposition of a 3–manifold there is a contact Morse function defining it, so a sequence of bypass attachments defines a contact Morse function. Thus Conjecture 15 would prove Conjecture 14.

Moreover a contact Morse function naturally defines an open book decomposition compatible with a contact structure with a page that is half of the critical submanifold $\{X \in \xi\}$. And vice versa an open book decomposition naturally defines a contact Morse function. This is done by fixing two pages to be the critical submanifold, and define the rest of the Morse function using the two halves of the S^1 factor. Thus essentially a contact Morse function contains the same information as an open book decomposition. So Conjecture 15 gives an easy proof for the (also not-written-down) connection of contact structures and open books.

8 "Overtwistedness" in higher dimension

Unlikely to the 3–dimensional case there is almost nothing known about higher dimensional contact structures and open books. There are two natural generalization of overtwistedness in the higher dimensional case. One of them is given by Giroux and the other one by Niederkruger [30]. I would like to prove that these two notions coincide.

Roughly speaking a *plastikstufe* or an *overtwisted family* with singular set S in contact manifold $(Y, \xi = \ker\alpha)$ is an embedded codimension two submanifold

$$\iota\colon D^2 \times S \hookrightarrow Y$$

that is maximally foliated by $\iota^*\alpha$ with leaves given by the characteristic foliation of an overtwisted disc (see Figure 1) times S. See Figure 6. Thus a plastikstufe has one closed leaf $\partial D^2 \times S$ and an S^1–family of leaves diffeomorphic to $(0,1) \times S$. For 3–dimensional contact manifolds S is a point and the above definition is naturally the original definition for an overtwisted manifold. The definition

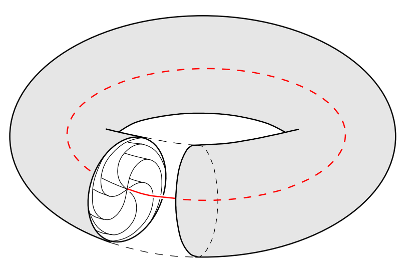

Figure 6: Plastikstufe with singular manifold S^1.

of Giroux uses open books. A 3–dimensional contact manifold is overtwisted if and only if it has an open book decomposition that is a negative stabilization of another open book decomposition.

Being the negative stabilization of an open book makes sense in higher dimension too. I intend to prove that these two definitions agree in higher dimension too.

Conjecture 16 *A contact structure contains a plastikstufe if and only if its open book is a negative stabilization.*

Approach

Both directions can be attacked by a generalization of Honda Kazez and Matic's partial open book decompositions [19]. Partial open book decompositions are generalizations of open book decompositions for contact manifolds with convex boundary. Since it is understood how to glue partial open books this definition should allow us to investigate the problem locally. I would like to construct a partial open book for a tubular neighborhood of a plastikstufe that is a stabilization of an other partial open book. With Etnyre we constructed such a partial open book for the neighborhood of an overtwisted disc in 3 dimension. This gives a new easier proof for the 3–dimensional version of the statement. For the other direction I would like to find a plastikstufe in the contact manifold gotten by the partial open book decomposition that is the negative stabilization of the trivial open book. This result is easy in 3–dimensions, but has been proven to be hard for higher dimensions.

Results From Prior NSF Support

The PI has not previously supported by NSF.

Broader Impacts

Maybe the best way to advertise Mathematics is by introducing it to young kids. Math Circles are Mathematical activities for elementary, middle and high school students. Usually the circle meet once or twice a week and the participants are presented challenging Mathematical problems of their level. Then they try to solve and discuss it amongst themselves or with the leader of the circle. This is one of the most rewarding kind of teaching, and these kind of activities have proven to be beneficial in the career of the participants. Math Circles have long traditions in Eastern European countries. Where most Mathematical Olympiad participants, and amazingly most Mathematics Professors in Mathematics at Universities grew up participating (several) Math Circles. This practice appears to be less widespread in the U.S., and seems to be key in encouraging the development of young Mathematicians. I have been participating in Math Circles since 1988 and have started to led one in 1997. I find it very important to make Mathematics available to young students, as these experiences can form their attitude towards Mathematics. Many of my former student I taught in their primary school years are doing Mathematics now. One former student of mine I am particularly proud of is Béla Rácz, who started his PhD studies in Topology at Princeton. Last year I taught Math Circles under the organization of MSRI one in Berkeley, CA and another one in Marine, CA. Part of my proposed activities would be to led a Math Circle in the Boston area.

A larger scale of people can be reached by expository articles. I have written two in a Hungarian journal, and plan to write another one on contact geometry. It is particularly important to write expository articles in languages that are spoken by few people (like Hungarian), as in teaching

Mathematics it is essential to have the base notions in that language. The Internet is also a good (and not yet fully used) way to advertise mathematics. As part of the Math Circle activities I accepted an invitation to give a web-talk for "Euclid Lab" a new Internet-based research opportunity for middle and high school students.

I am also working on involving graduate students in research. I gave several talks specifically aimed at graduate students, I also gave a mini course on Heegaard Floer homologies at the University of Cape Town. I organized seminars both at Rényi Institute and at MSRI. I led problem sessions on the Summer Graduate Workshop on Symplectic and Contact Geometry and Topology at MSRI in 2009. I have been asked to led Problem Sessions at the Trimester on Contact and Symplectic Topology in 2011 in Nantes. One of my proposed activities is to give graduate-level seminars on low dimensional topology at MIT and other Universities in the U.S. and abroad.

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Pacific Journal of Mathematics, Vol.239 1:157–177, 2009. http://arxiv.org/abs/0806.1436
Cs. Szabó and V. Vértesi: **The complexity of the word-problem for finite matrix rings.**
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Synergistic Activities

2009 **Seminar on higher dimensional open books**, organizer, MSRI
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