LEGENDRIAN AND TRANSVERSE INVARIANTS IN HEGAARD FLOER HOMOLOGY

Joint work w\ J.A. Baldwin & D.S. Vela-Vick

Legendrian and Transverse Knots

Remember: a contact structure is a totally nonintegrable planefield ξ on a 3-manifold Y.

a knot in (Y, ξ)

$T_pL \in \xi$
$\forall p \in L$

Legendrian knot

$T_pT \pitchfork \xi$
$\forall p \in T$

transverse knot

every knot can be put both in Legendrian and transverse position

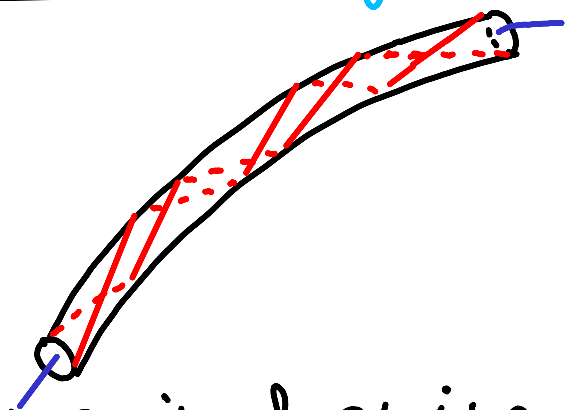
$L \longrightarrow T(L)$ transverse push off

Legendrian approximation (not unique) $\longleftarrow T$

Thm: transverse knots / transverse isotopy $\longleftrightarrow$ Legendrian knots / Legendrian isotopy + negative stabilization

Legendrian Invariant in Sutured Floer Homology

Fact A Legendrian knot has a standard neighborhood with convex boundary having a 2-component dividing curve each of which represents the Thurston-Bennequin framing



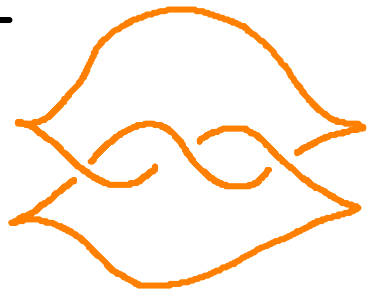
$L \hookrightarrow (Y, \xi)$ Legendrian knot N_L its standard neighborhood
$\rightsquigarrow (Y - N_L, \xi|_{Y-N_L})$ is a contact 3-manifold with boundary

$$c(L) := \hat{c}(\xi|_{Y-N_L}) \in SFH(-(Y-N_L), \Gamma_{\partial N_L})$$

- $c(L)$ is an invariant of L

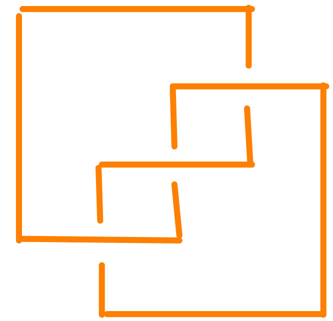
Other Invariants in Heegaard Floer Homology

Combinatorial:



rotate 45°

and introduce more corners



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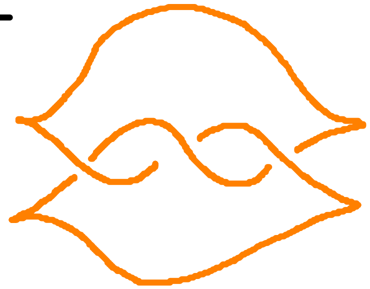

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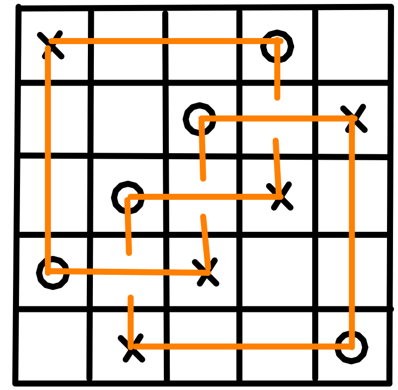

include in a grid

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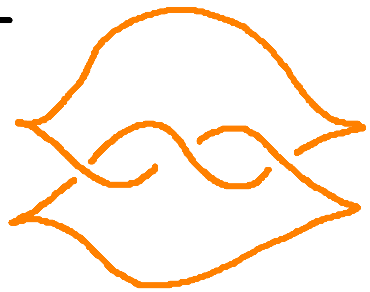

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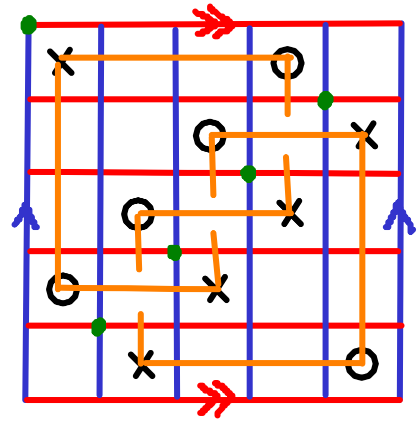

multipointed Heegaard diagram on a torus for the knot type of L

Thm (Ozsváth - Szabó - Thurston)

- defines an element in $\widehat{HFK}(-Y,K)$: $\lambda(L)$ an invariant for L

Loss invariant:

L can be put homologically nontrivialy on the page of an OB

Thm (Lisca - Ozsváth - Stipsicz - Szabo)

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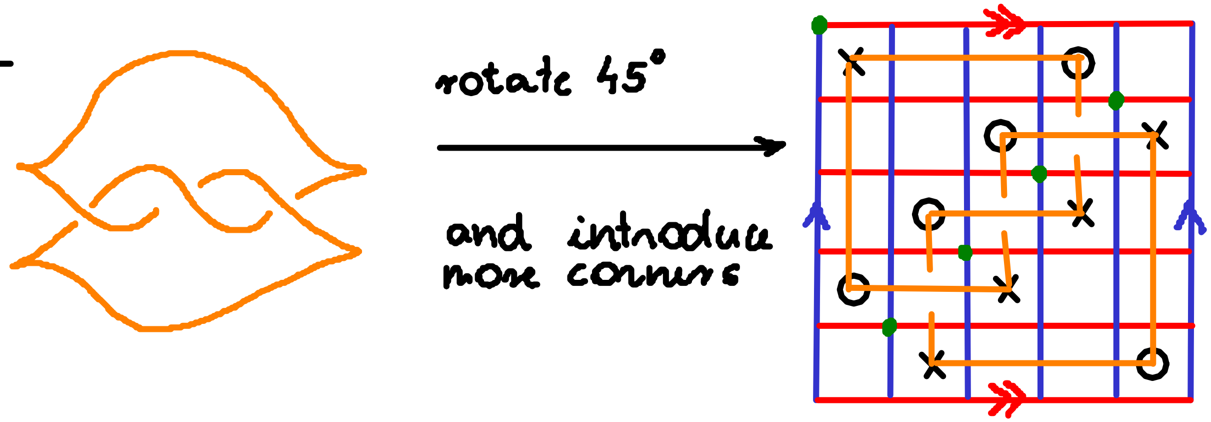

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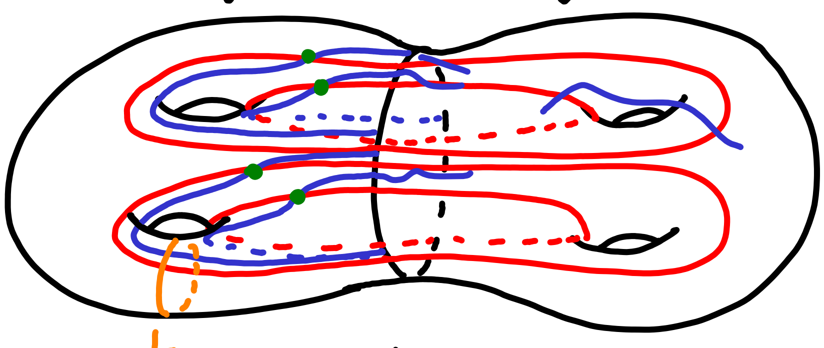

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Properties of These Invariants

Def L <u>loose</u> if its complement is OT
L is <u>exceptional</u> or <u>non-loose</u> otherwise

		$c(L)$	$\mathcal{L}(L)$	$\lambda(L)$
		in $SHF(-Y\setminus N(L),\Gamma)$	in $HFK(-Y,K)$	in $HFK(-S^3,K)$
L loose		0 HKM	0 LOSS	N.A.
complement of L contains Giroux torsion		0 HKM	0 Vela-Vick, Stipsicz-V	N.A
stabilisation	L^+	$SHF(-Y\setminus N(L),\Gamma)$ $\downarrow$	0	0
	L^-	$SHF(-Y\setminus N(L^-),\Gamma)$	$\mathcal{L}(L)$ LOSS	$\lambda(L)$ O-Sz-T

Cor: Both $\mathcal{L}(L)$ and $\lambda(L)$ defines a transverse invariant by:

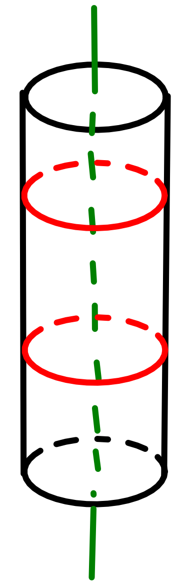
$$\vartheta(T) = \lambda(L) \qquad \text{if } T = T(L)$$
$$\tau(T) = \mathcal{L}(L)$$

Connections between these Invariants

Thm (Stipsicz - V)

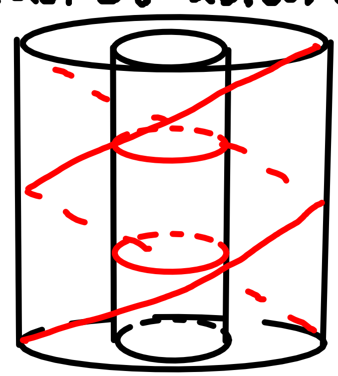
$$\begin{tikzcd} SHF(-Y\setminus N(L),\Gamma) \arrow[r, bend left, "+"] \arrow[r, bend right, "-"'] & HFK(-Y,K) \end{tikzcd}$$

$$\begin{tikzcd} & \mathcal{L}(L) \\ c(L) \arrow[ur, mapsto, bend left] \arrow[dr, mapsto, bend right] & \\ & \mathcal{L}(\overline{L}) \end{tikzcd}$$



idea: $HFK(-Y,K)$ is a sutured Floer homology

Use HKM-map for a contact structure filling:



Thm (Baldwin - Vela-Vick - V)

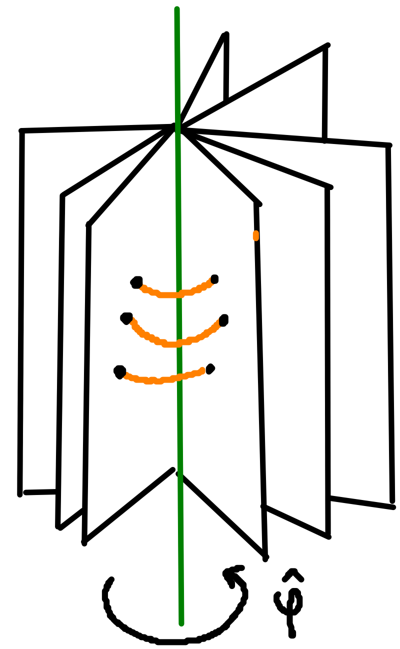
There is an invarian for transverse knots: $t(T)$

$$\vartheta(T) = t(T) = \mathcal{T}(T) \quad \text{for any transverse knot in } (S^3,\xi_{st})$$

Cor: $\lambda(L) = \mathcal{L}(L)$ for any Legendrian knot in (S^3,ξ_{st})

Towards the Definition of $t(T)$

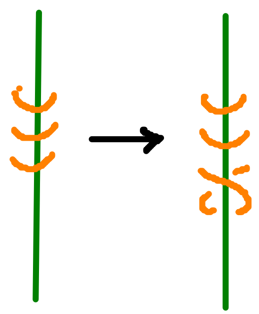
Thm: (Pavelescu) Given an OB (S,φ) then any transverse knot can be isotoped to be transverse to the pages: B



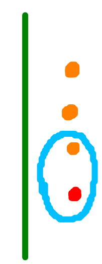
$B \rightsquigarrow \hat{\varphi} : (S, \{n \text{ pts}\}) \circlearrowleft$ lifted monodromy

Thm (Pavelescu +) Two lifted monodromies define transverse isotopic transverse knots if they are related by a finite sequence of the following moves

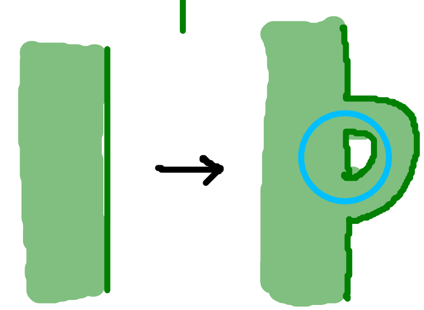
- positive braid stabilisation



or adding an extra pt & composing the lifted monodromy w/ ½ Dehn twist:



- positive stabilization of the open book



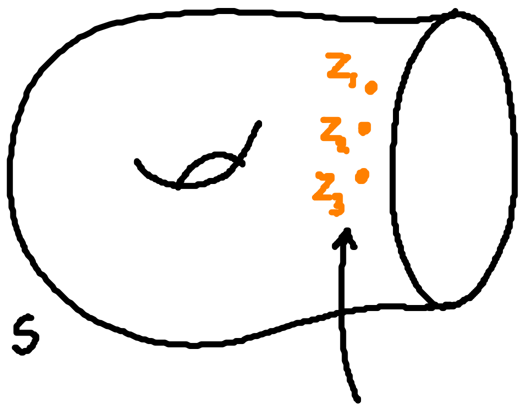
- conjugation of the braid word

A New Transverse Invariant

Given T

Pick an OB (S,φ) compatible w/ ξ and make T transverse to the OB

$\rightsquigarrow \hat{\varphi}$ a lifted monodromy



intersection of T with a page

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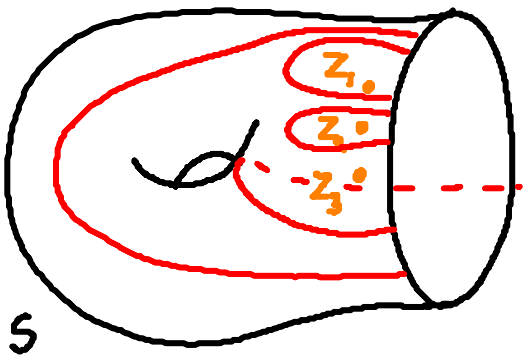

pick arcs that separate the z_i's into discs

A New Transverse Invariant

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pick arcs that separate the z_i's into discs

$\rightsquigarrow$ • defines an element in $HFK(-Y, K)$

! and it turns out to be an INVARIANT of transverse knots!: $t(T)$

Moreover:

Thm: For transverse knots in (S^3, ξ) $t(T)$ is the bottommost element of $HFK(-Y, K)$ with respect to the filtration given by the page.

Cor (Baldwin – Vela-Vick - V)
$\vartheta(T) = t(T) = \mathcal{T}(T)$ for any transverse knot in (S^3, ξ_{st})

Thanks for your attention!