PÉTER

VÉRTESI

$$\begin{array}{r} 2011 \\ -1941 \\ \hline 70 \end{array}$$

Some Theorems of P. Vértesi

Thm (Erdős – Vértesi) For any system of nodes on $[-1,1]$
There is $f \in C[-1,1]$ such that

$$\varlimsup_{n\to\infty} |L_n(f,X,x)| = \infty \quad \text{almost everywhere in } [-1,1]$$

Moreover $\{x : \varlimsup_{n\to\infty} |L_n(f,X,x)| = \infty\} \subseteq [-1,1]$ is of second category.

Thm (Vértesi) For any system of nodes on the unit circle line Γ
there is $f \in AC$ such that

$$\varlimsup_{n\to\infty} |L_n(f,X,x)| = \infty \quad \text{almost everywhere on } \Gamma$$

Thm (Vértesi) For any system of nodes on $[-1,1]$ and $w(t)$ with

$$\varliminf_{t\to 0} w(t)|\log(t)| = \infty$$

there is $f \in C(w)$ such that
$\{x : \varlimsup_{n\to\infty} |L_n(f,X,x)| = \infty\} \subseteq [-1,1]$ is dense and of second category.

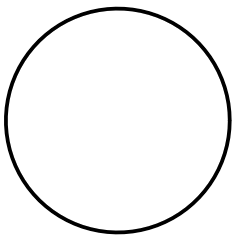
Thm (Vértesi) For any system of nodes on $\mathbb{R}$
There is $f \in \tilde{C}$ such that

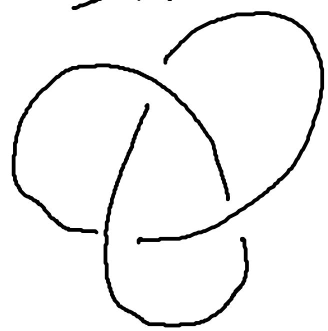
$$\varlimsup_{n\to\infty} |L_n(f,\Theta,\vartheta)| = \infty \quad \text{almost everywhere on } \mathbb{R}$$

Moreover $\{\vartheta : \varlimsup_{n\to\infty} |L_n(f,\Theta,\vartheta)| = \infty\} \subseteq \mathbb{R}$ is dense & of second category.

Smooth Knots

$$S^1 \underset{\text{smooth embedding}}{\xhookrightarrow{\ \iota\ }} \mathbb{R}^3 \xrightarrow{\ \pi\ } \mathbb{R}^2$$

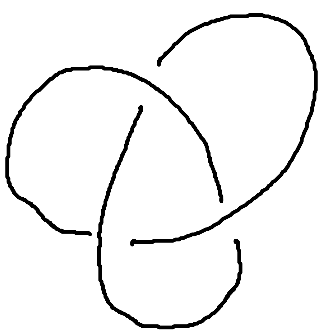




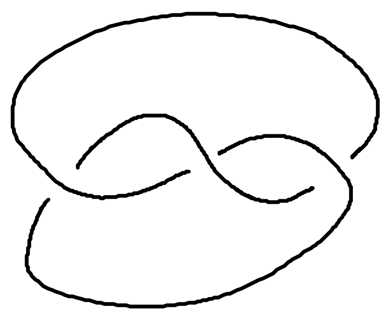
projection w/ over/under info

Isotopy: deforming ι smoothly in space (every ι_t is a smooth embedding)

e.g.:



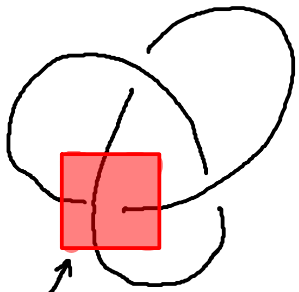
$\overset{\text{isot}}{\approx}$



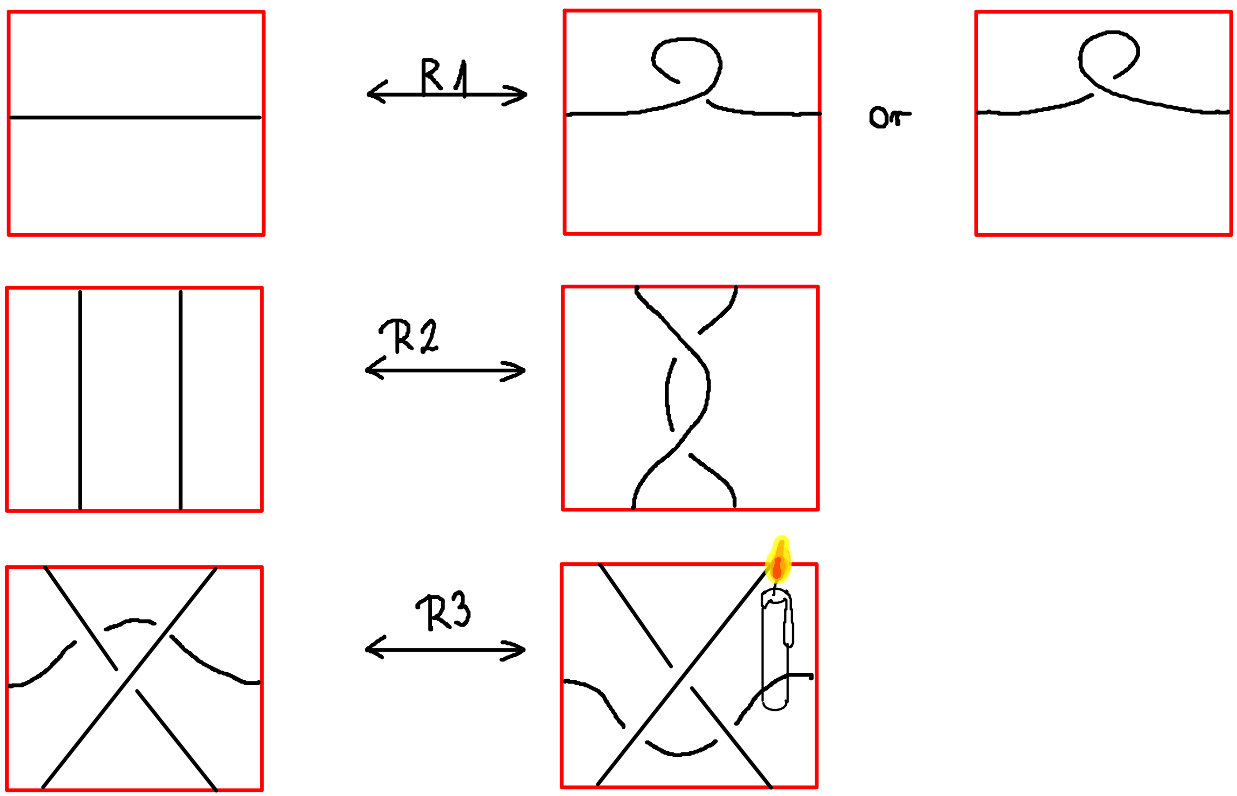
Goal: Classify knots up to isotopy

Isotopy in the Projection

Local moves



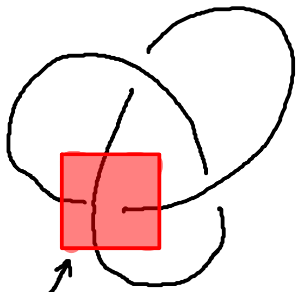
Change the projection in the box, while leaving the rest unchanged



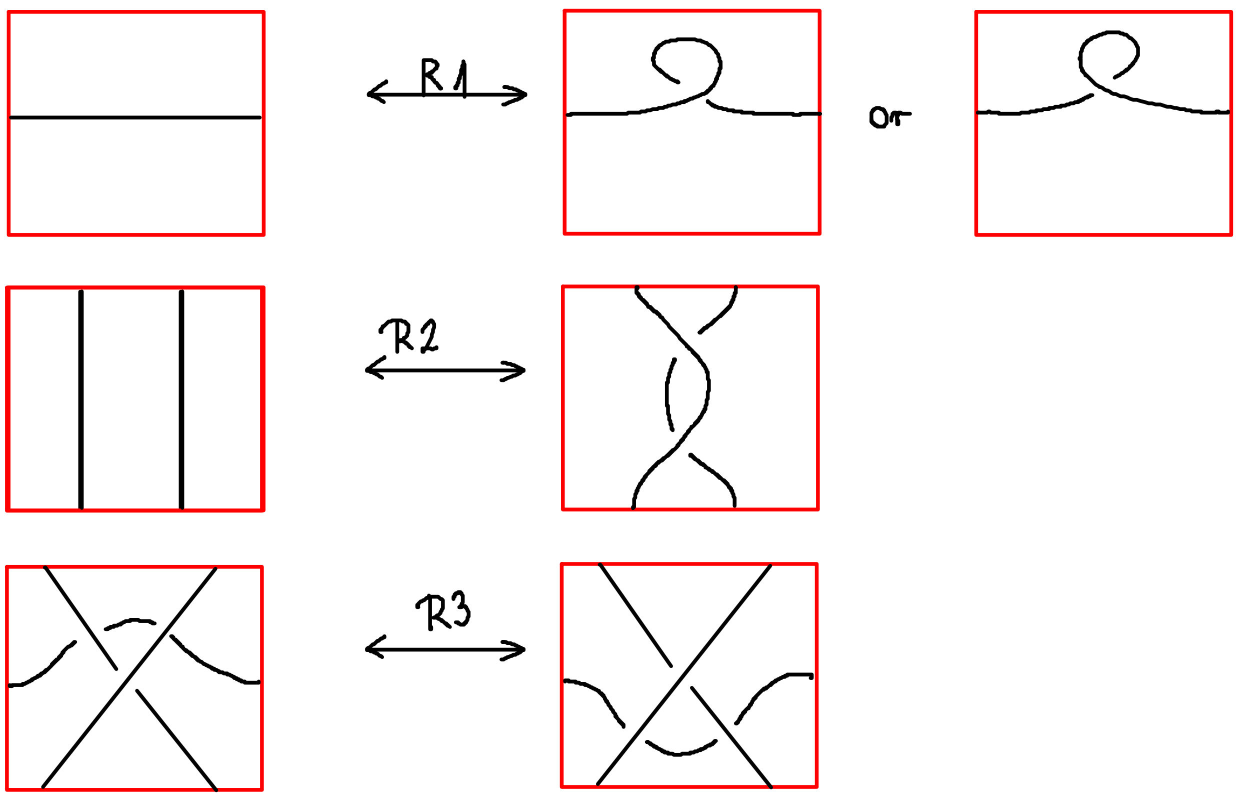
¿ other local moves
global moves ?

Isotopy in the Projection

Local moves

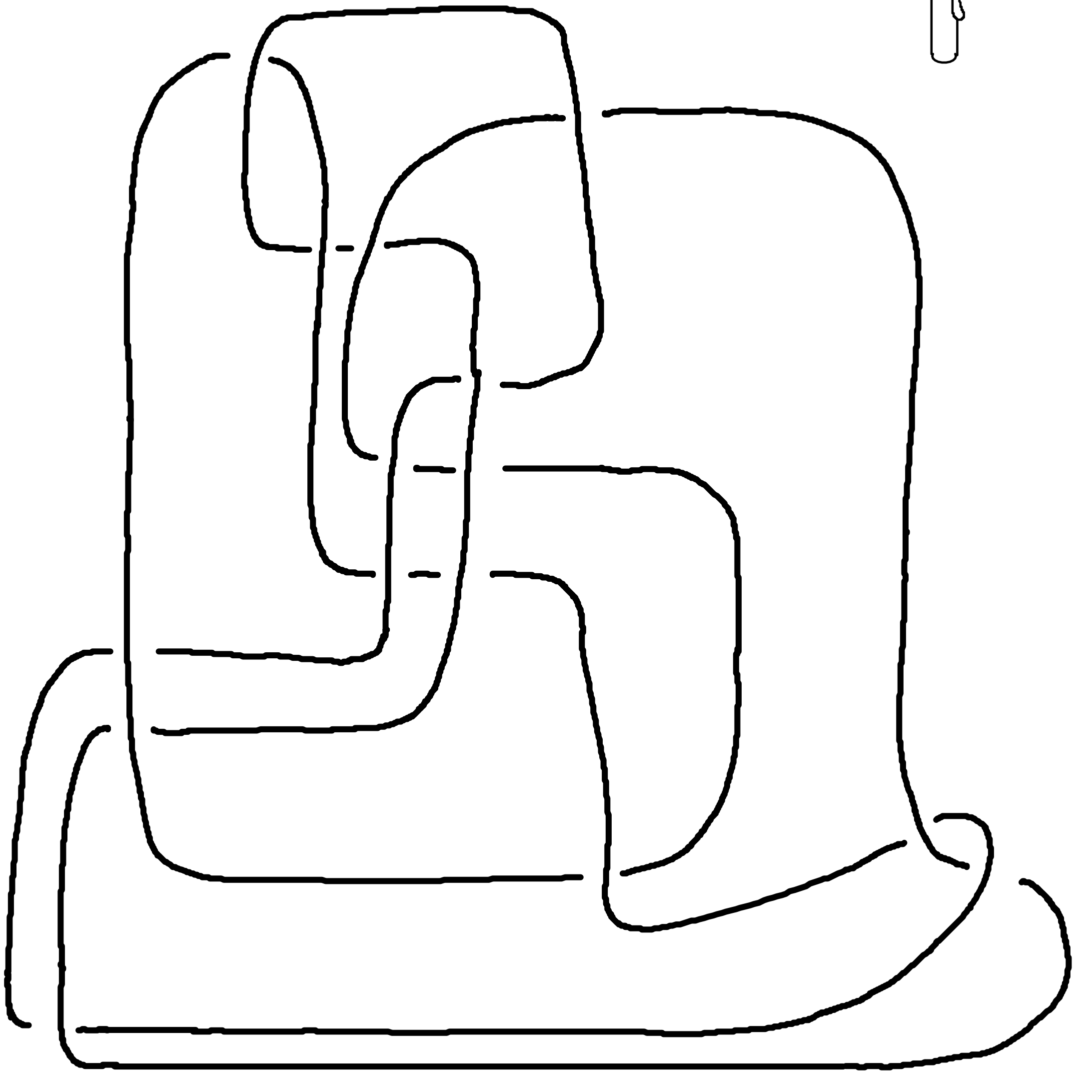


Change the projection in the box, while leaving the rest unchanged

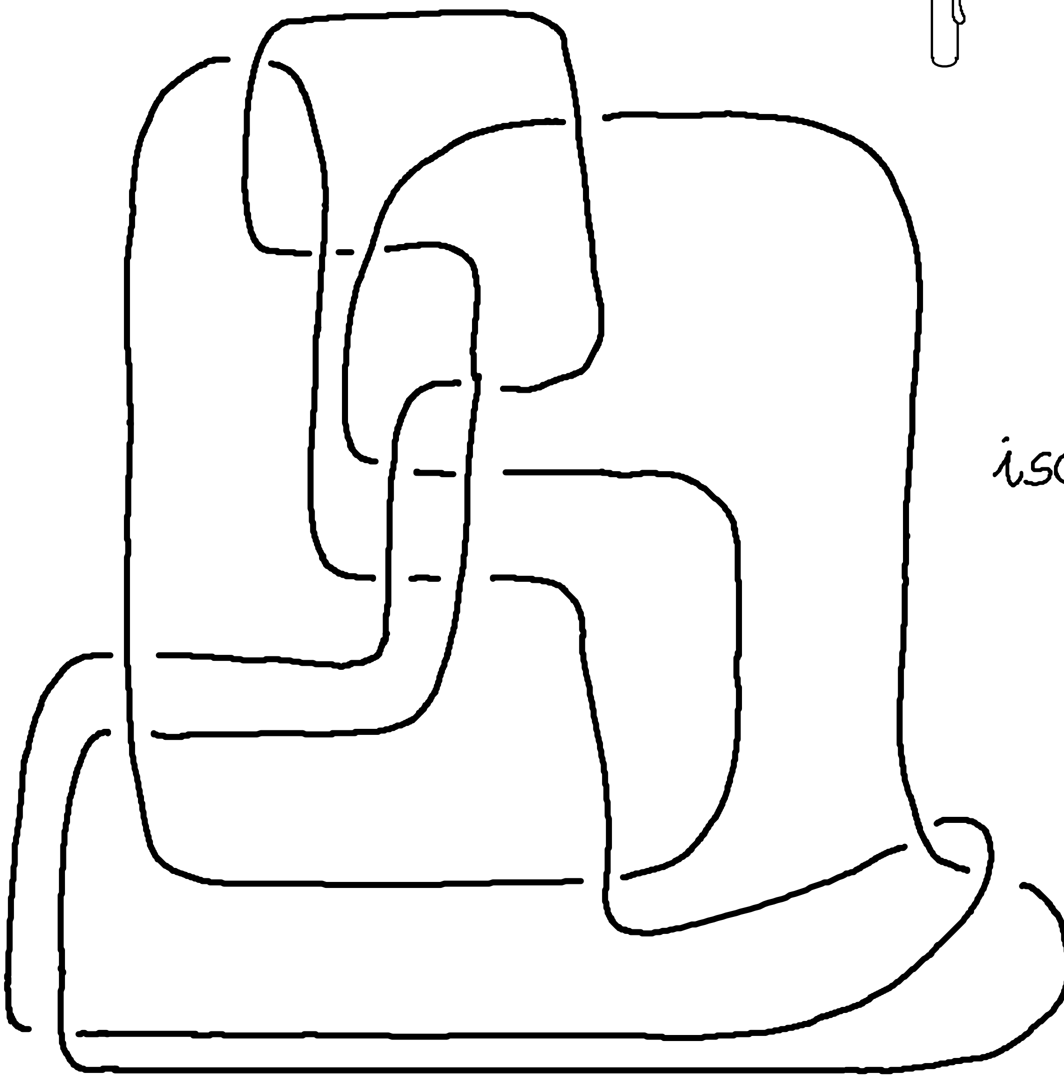


Thm (Reidemeister) Two projections correspond to isotopic knots if they are related by a sequence of the above moves and planar isotopies.

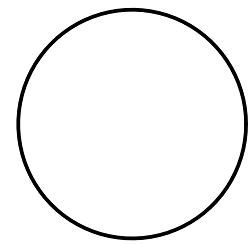
EXAMPLE: MORWEN THISLEWAITHE



Example: Morwen Thislewaithe



isotopic to the unknot:

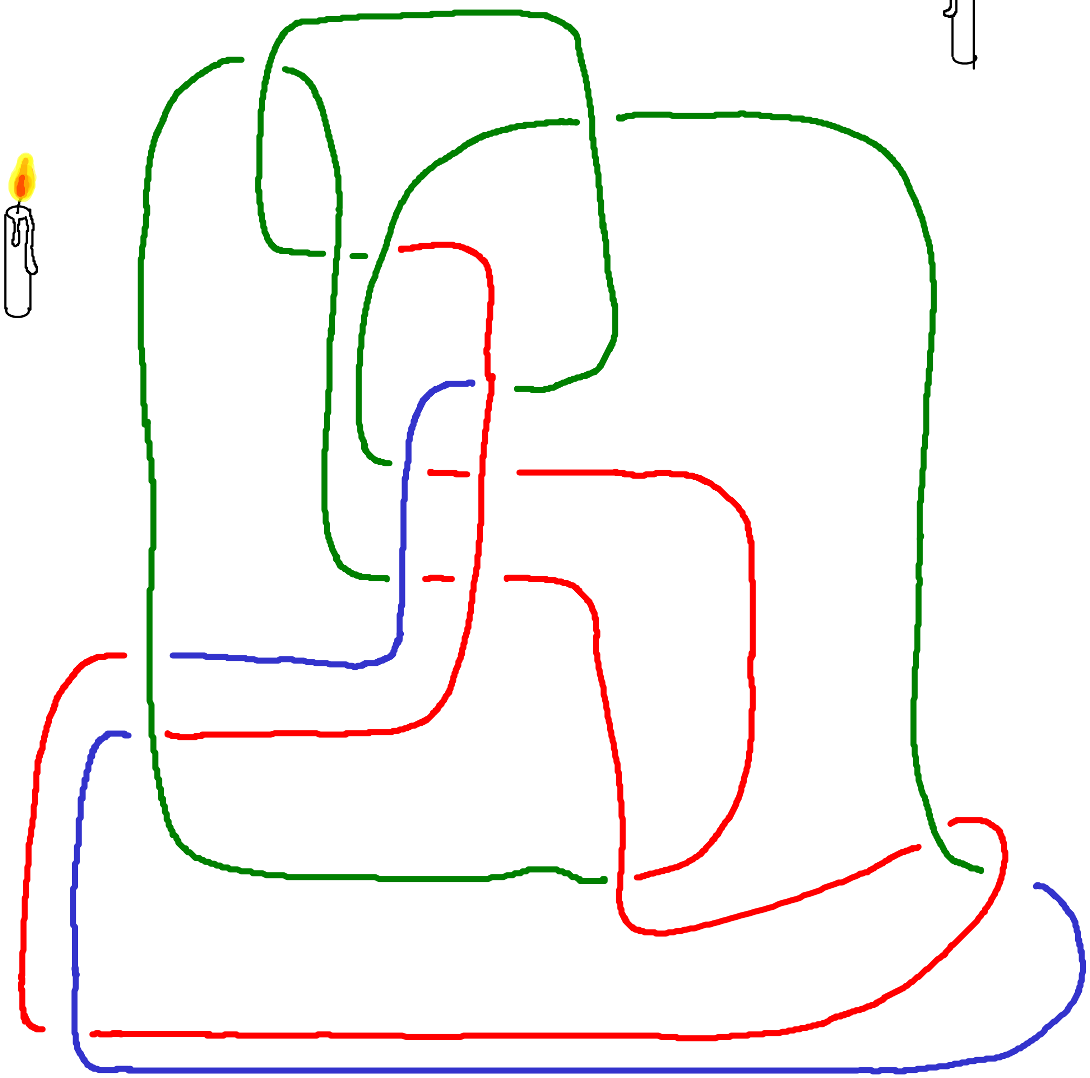


But: hard to show

Question: Can we always find the isotopy if it exists?

What can we do if there is no isotopy?

A Knot Invariant: 3-coloring



coloring of the arcs of the projection w/ 3 colors

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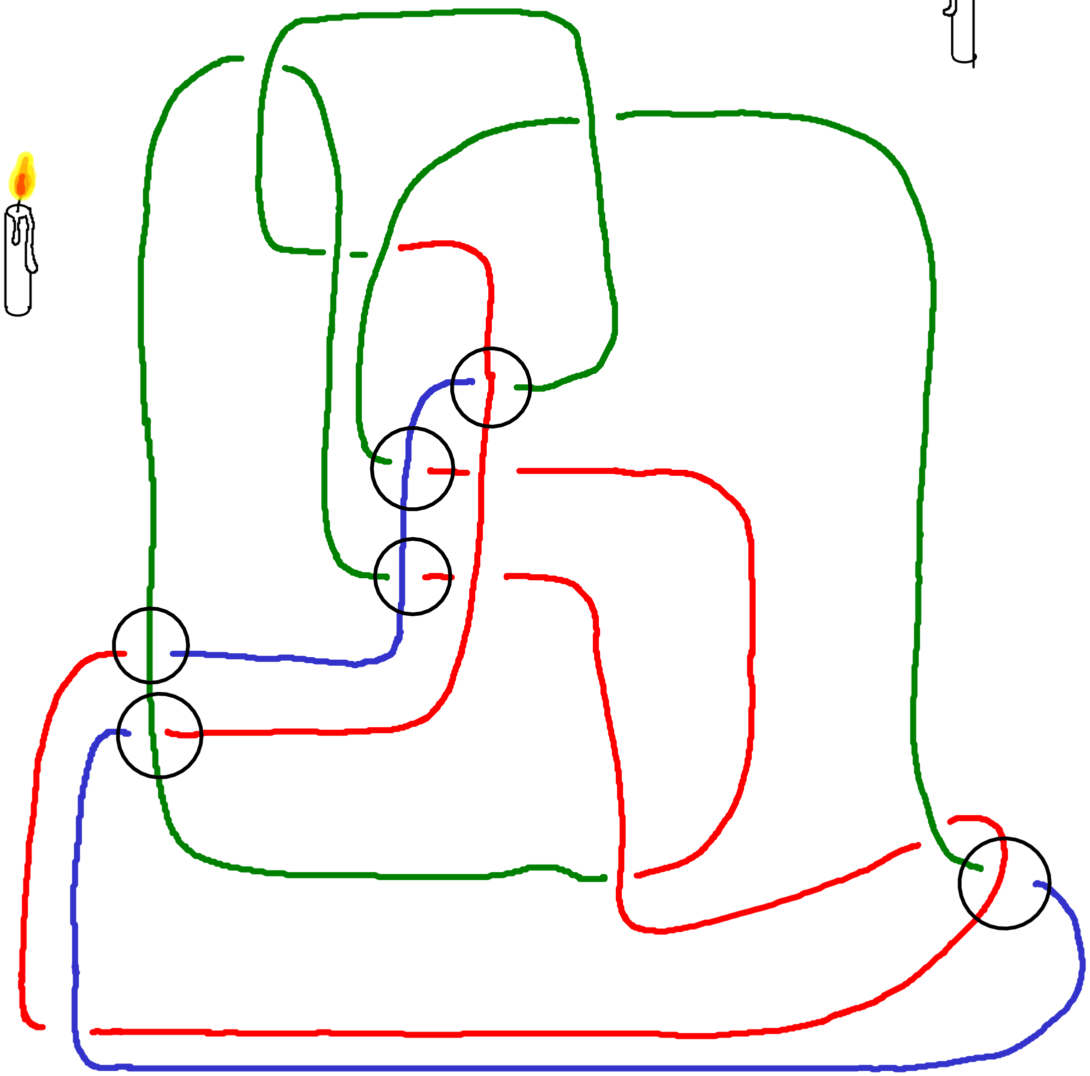
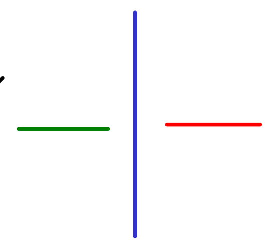


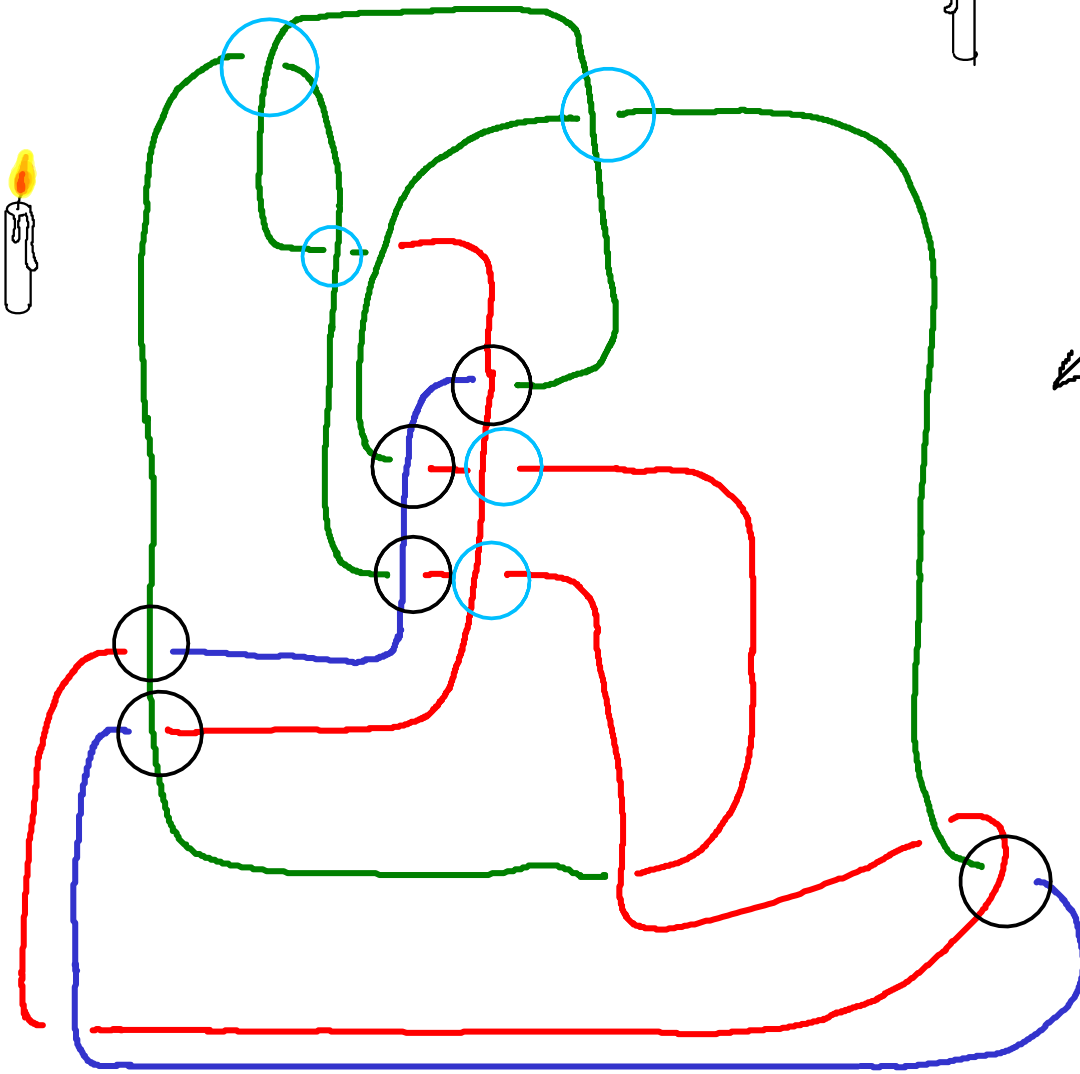
coloring of the arcs of the projection w/ 3 colors

such that each crossing is either



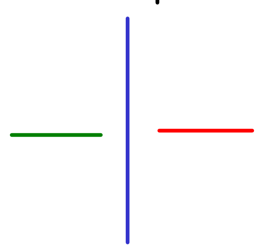
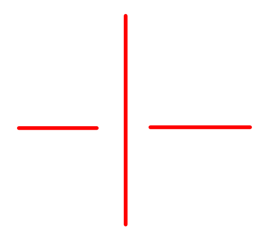
· tricolor

A KNOT INVARIANT: 3-COLORING

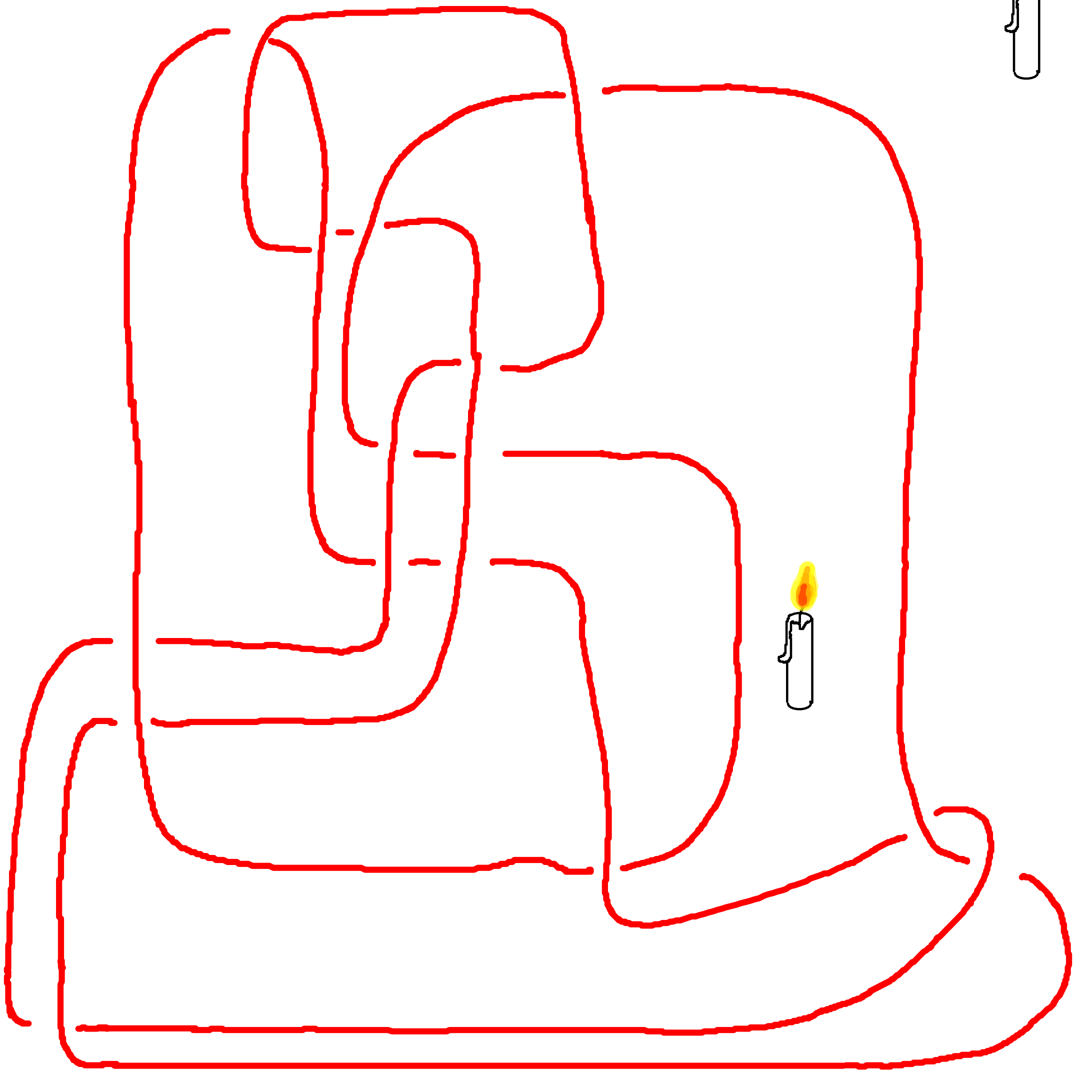


Not a 3-coloring

coloring of the arcs of the projection w/ 3 colors
such that each crossing is either

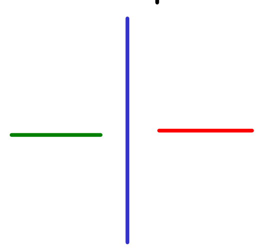
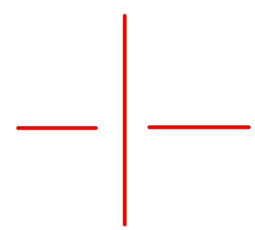
- tricolor 
- monochromatic: 

A KNOT INVARIANT: 3-COLORING



Contains more than one color!

coloring of the arcs of the projection w/ 3 colors such that each crossing is either

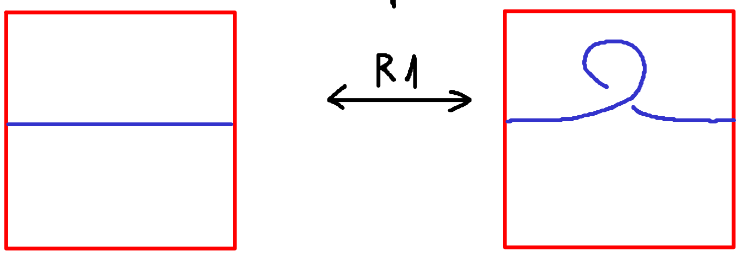
- tricolor 
- monochromatic: 

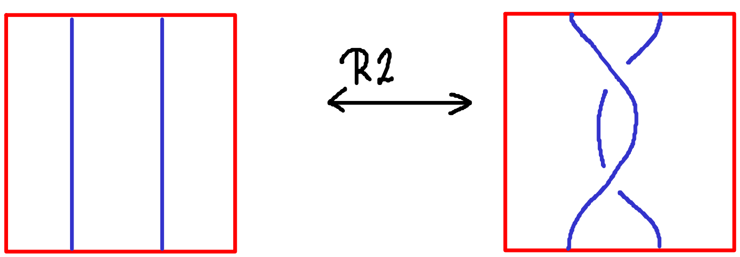
Invariance

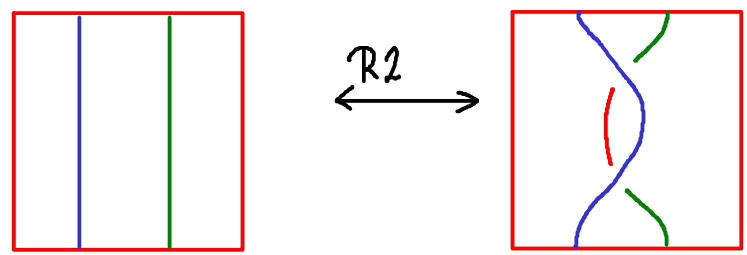
So far 3-colorability is a property of a projection (not an isotopy class..)

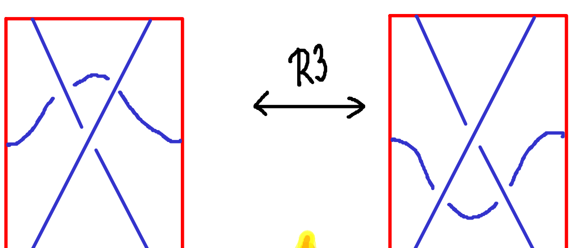
Thm (Reidemeister) Two projections correspond to isotopic knots if they are related by a sequence of the above moves and planar isotopies.

Need to see that 3-colorability is invariant under the above moves

- planar isotopies ✓
- 

✓
- 

or


- 

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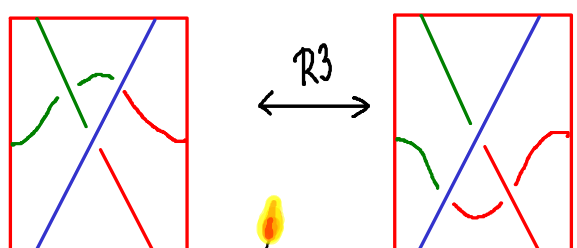

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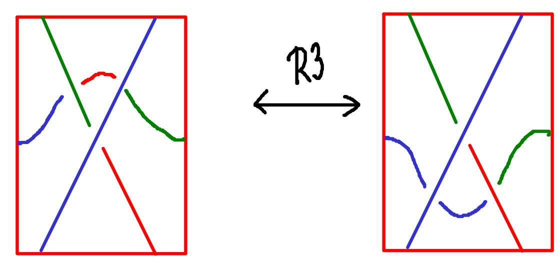
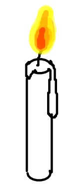
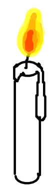


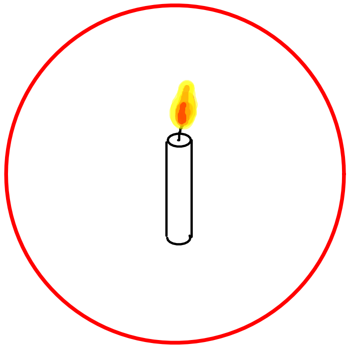
EXAMPLE: A KNOT THAT CANNOT BE UNTIED

We have seen that 3-colorability is isotopy-invariant.

Is the unknot 3-colorable?







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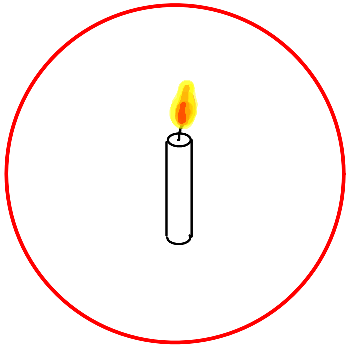

NO

Is there a 3-colorable knot?

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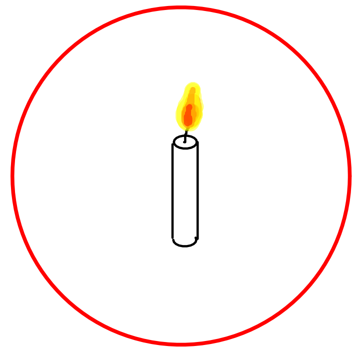

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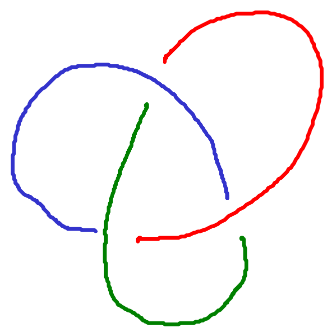

Thus the trefoil knot cannot be untied
(not isotopic to the unknot)

BOLDOG

SZÜLETÈSNAPOT

APU!