Main Thm 1

(1) $\exists!$ gp hom $\mathrm{Art}_K : K^\times \longrightarrow W(K^{LT}/K)$ s.t.

(a) $\forall$ unif π_K of K, $\mathrm{Art}_K(\pi_K)|_{K^{ur}} = \mathrm{Frob}_K$.

(b) $\forall K'$, $\underbrace{K \subset K'}_{fin} \subset K^{LT}$, $\mathrm{Art}_K(N_{K'/K}(K'^\times))\big|_{K'} = \{1\}$.

pf of uniqueness

$$\mathrm{Art}_K = \mathrm{Art}_K'$$

$\Leftarrow$ $\mathrm{Art}_K(x) = \mathrm{Art}_K'(x)$, $\forall x \in K^\times, v(x) = 1$. ($\because$ Such x generate $K^\times$ as a group.)

$\overset{\Leftarrow}{(a).}$ $K^{LT} = K_x \cdot K^{ur}$ $\quad \mathrm{Art}_K(x)|_{K_x} = \mathrm{Art}_K'(x)|_{K_x}$, $\forall x \in K^\times, v(x) = 1$.

$\Leftarrow$ $\mathrm{Art}(x)|_{K_x^m} = \mathrm{Art}_K'(x)|_{K_x^m}$, $\forall$ " , $\forall m \geq 1$.

$K_x = \bigcup_{m\geq 1} K_x^m$

The last equality is deduced from two facts:

① $\begin{cases} \mathrm{Art}_K(N_{K_x^m/K}(K_x^{m\times}))\big|_{K_x^m} = \{1\} \\ \mathrm{Art}_K'(\quad " \quad)\big|_{K_x^m} = \{1\}. \end{cases}$ by (b).

② $\forall m \geq 1$, $\forall x \in K^\times, v(x) = 1$, $x \in N_{K_x^m/K}((K_x^m)^\times)$

($\because$ $K_x^m = K(\mu_{f,m}^x) = K(\alpha)$. $\forall \alpha \in \mu_{f,m}^x = \{\text{roots of } f_m^x\}$.

$$f_m^x = \prod_{\beta \in \mu_{f,m}^x} (X - \beta) = \prod_{\sigma \in G(K_x^m/K)} (X - \sigma(\alpha)).$$

Since $f_m^x = f_m/f_{m-1}$ has const term $x^{\varphi^{m-1}} = x$, ([Y] 4.4.(ii))

$$N_{K_x^m/K}(-\alpha) = x.$$

So $x \in N_{K_x^m/K}((K_x^m)^\times)$)