18.031

Day 2

Plan

- Another oscillator
- Resonant tuning
- Bode & Nyquist plots
- Transfer/system function
- Short note on Block diagrams (see class 2 reading)
- Pole diagrams
- Mascot

Recall: gain, complex gain & phase lag

Given: $P(D)x = Q(D)e^{i\omega t}$. Gen. sol.: $x_{gen} = x_h + x_p$

$$G(\omega) = \frac{Q(i\omega)}{P(i\omega)}$$

complex gain

$$x_p = Re(z_p) = |G|\cos(\omega t - \phi)$$

gain phase lag

$$g(\omega) = |G(\omega)| \quad , \quad \phi(\omega) = -\arg(G(\omega))$$

Why not care about homogeneous part of solution? => Because it's a transient!

Stability

When homogeneous part dies off exponentially: STABLE system

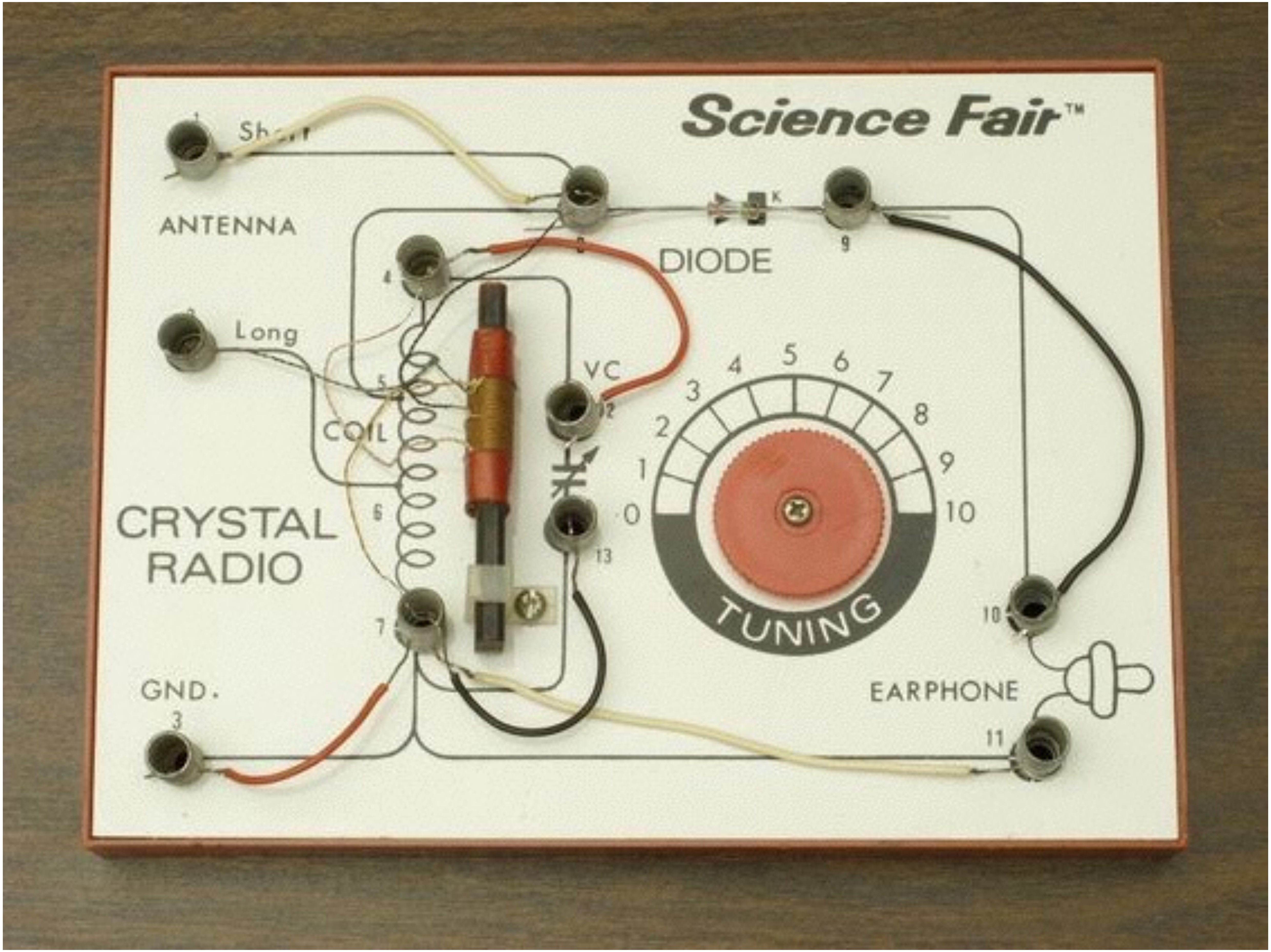
What makes system stable? E.g:

$$\text{Roots of } P(s): s = \frac{1}{2}(-1 \pm i\sqrt{7})$$

$$x_h = e^{-t/2}\left[a_1 \cos(\sqrt{7}t/2) + a_2 \sin(\sqrt{7}t/2)\right]$$ transients

Re(s)<0 for all roots s of P(s) <=> STABLE

Sparton radio Model 652X from 1940-1942



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Modeling an AM radio

Basic process of modeling a system:

(1) *Draw a diagram of the system.*

(2) *Identify and give symbols for the parameters of the system.*

(3) *Declare the input signal and the system response.*

(4) *Write down a differential equation relating the input signal and the system response, using Newton's "$F = ma$" in the mechanical case or Kirkhoff's laws in the electrical case.*

(5) *Rewrite the equation in standard form.*

RLC and mechanical systems

Mechanical		Electrical	
displacement	x	voltage drop	V
mass	m	inductance	L
damping constant	b	resistance	R
spring constant	k	1/capacitance	$1/C$

$$m\ddot{x} + b\dot{x} + kx = b\dot{y} \qquad L\ddot{V}_R + R\dot{V}_R + (1/C)V_R = R\dot{V}$$

Mathlet

$$m\ddot{x} + b\dot{x} + kx = b\dot{y} \qquad L\ddot{V}_R + R\dot{V}_R + (1/C)V_R = R\dot{V}$$

We already know:

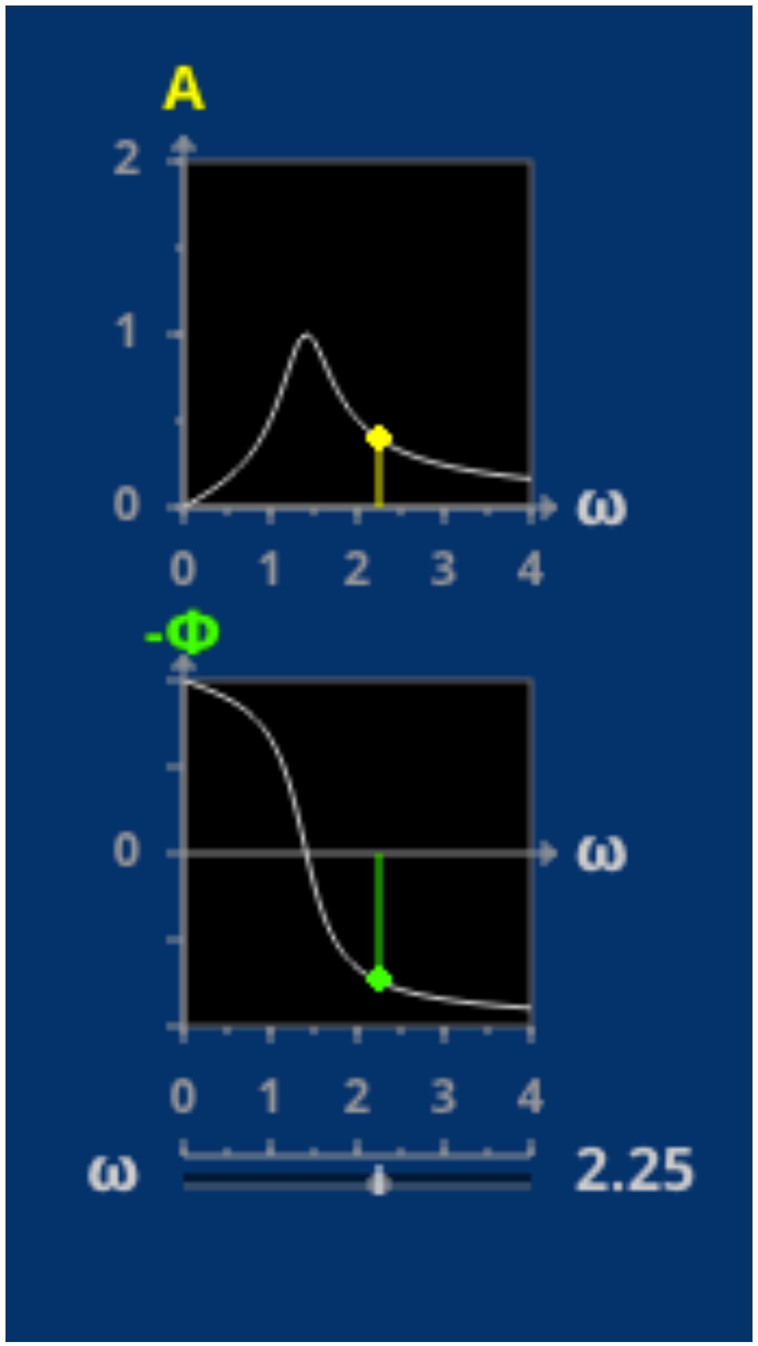
Maximum gain g=1 for any fixed b,k

Maximum gain at $\omega = \omega_r \sim \sqrt{k/m}$ (indep. of b!)

Can tune radio to "target channel" by changing k (or m)!

Would be nice to "visualize" how system reacts to different frequencies in a plot!

Mathlet: Bode plots



Plot of (real) gain $g(\omega) = |G(\omega)|$

Plot of phase *gain* $-\phi = \mathrm{Arg}(G(\omega))$

Often: double-logarithmic axes for gain plot (“dB”)!

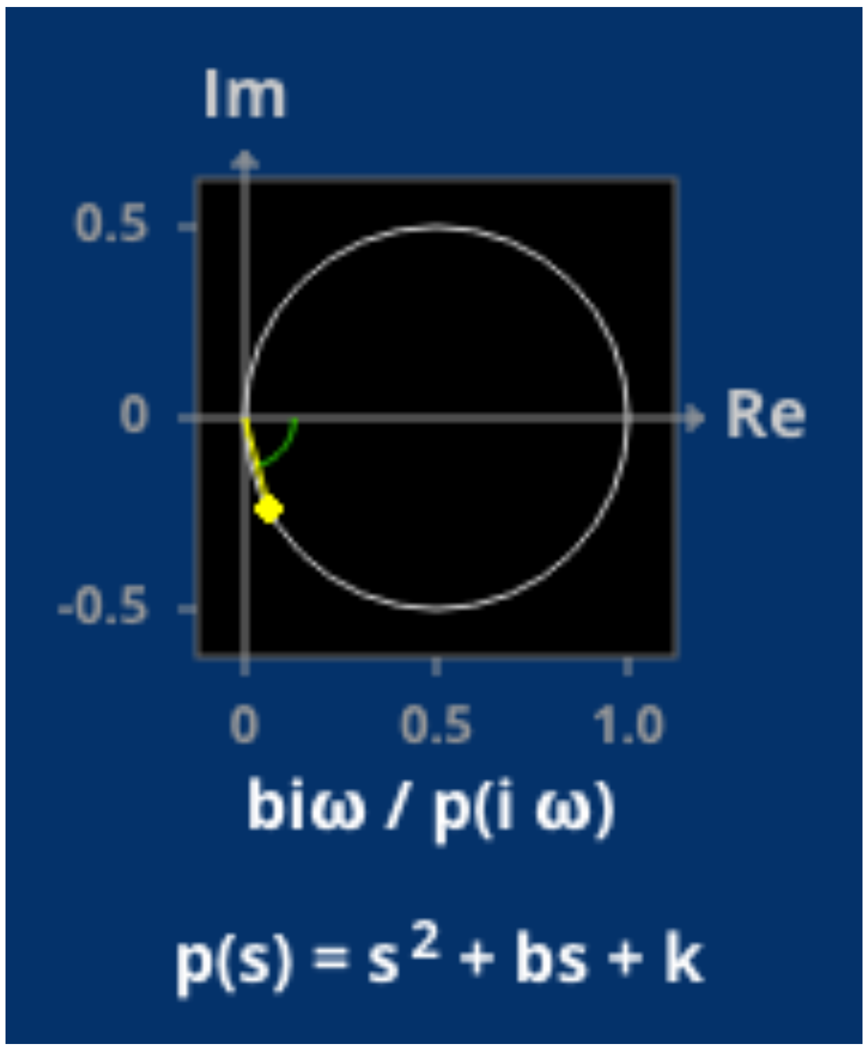
AKA: **Amplitude-response curve**

Activity: Verify our observations mathematically!

Here are the observations you have made about this system, translated into the notation of the RLC circuit. The Mathlet fixes $m = 1$, which corresponds to setting $L = 1$.

1. The maximum gain is 1, independent of the values of R, and C.
2. The maximum gain occurs at $\omega_r = \sqrt{1/C}$, independent of R.
3. The phase lag at $\omega = \omega_r$ is $\phi(\omega_r) = 0$, while for $\omega < \omega_r$ the phase lag is *negative*: the system response appears to run *ahead* of the input signal.
4. As R decreases, the pass-band narrows.

Mathlet: Nyquist plots



Plot of complex gain in complex plane

Yellow: magnitude of G, ie. real gain.

$$g(\omega) = |G(\omega)|$$

Green: $-\phi = \mathrm{Arg}(G(\omega))$

=> phase gain

Curve parameterized by w!

System function

So far: $P(D)x = Q(D)e^{i\omega t}$. → $G(\omega) = \frac{Q(i\omega)}{P(i\omega)}$ complex gain

We only considered sinusoidal input so far. What about more general exponential inputs like e^{st} with s complex?

E.g.: $s = -1/10 + i\pi$ → input: $e^{-t/10}\cos(\pi t)$

But derivation of ERF (see notes) also works if s is complex! So:

System $P(D)x = Q(D)e^{st}$ system response $x_p = H(s)e^{st}$

$$H(s) = \frac{Q(s)}{P(s)}$$ *system function* or *transfer function*

System function & gain

$$H(s) = \frac{Q(s)}{P(s)} \quad \Longleftrightarrow \quad G(\omega) = \frac{Q(i\omega)}{P(i\omega)}$$

System function extends concept of gain to any complex s.
“Gain is system function along imaginary axis s=iw.”

Powerful tool to understand a system’s behavior to more complicated inputs (in fact, to ALL inputs - later). See also Block diagrams

How can get a qualitative feeling for H(s)?

=> Idea: Graph lH(s)l in the complex plane!

Tuned mass damper