18.031

Day 1

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Preliminaries

- Course website: http://math.mit.edu/~stoopn/18.031/
- Two psets, one 1h exam (50/50 grade)
- Exam: 1h, during class, Thursday Feb. 2, 3-4pm
- Pass/fail
- Pset 1 available on Stellar.
- 18.03 knowledge assumed
- Class readings on website

Course contents

- Define the notions of stability, gain, phase lag, frequency response and system function for LTI systems.
- Analyze LTI systems in the frequency domain
- Transfer function and block diagrams
- Interpret the pole diagram of a system in terms of stability, gain and resonance
- Laplace transform of a function
- Inverse Laplace transform to compute the unit impulse response of a system modeled by a differential equation.
- Examples from electrical and mechanical engineering.

System control…

Day 1: 18.03 Review

1. find a basis of solutions to a homogeneous linear constant coefficient ODEs, in terms of exponentials and sinusoids, and determine whether the equation is stable or not;
2. find a particular solution to a linear constant coefficient ODE with right hand side made up of exponentials and sinusoids;
3. use the principle of superposition to find the general solution in terms of these first two procedures;
4. model mechanical systems using differential equations, using the language of input signals and system response and the standard form $P(D)x = Q(D)y$ in terms of characteristic polynomials.
5. determine the complex gain of a system and extract from it the gain and phase lag.

Mathlet Amplitude & Phase 2nd order II

Activity 1

Open Mathlet AMPLITUDE AND PHASE: SECOND ORDER II

1. What does the blue curve represent?
2. What does the yellow curve represent?

The *gain* of this system is

$$\text{gain} = \frac{\text{amplitude of system response}}{\text{amplitude of input signal}}.$$

In this Mathlet, the amplitude of the input signal is fixed at 1, so the gain equals the amplitude of the sinusoidal system response.

Activity 1

3. Take $b = 1.0$, $k = 2.0$, and $\omega = 1.00$, and measure the gain. (Note: you can set the sliders to certain values by clicking on the hashmarks.)

4. Stay with these values of b and k, but vary ω. What is the maximum gain you observe for this system?

5. Now pick some other value of b and k, and vary the input frequency ω. What is the maximum gain in these cases? Care to formulate a general conjecture?

6. The angular frequency at which the gain is maximal is the "resonant angular frequency" ω_r. Set $b = 1.0$ and $k = 1.0$. Measure the value of ω_r. Same question for $k = 2.0$ and $k = 4.0$. Please suggest a formula for ω_r in terms of k, for this value of b.

7. Now select at least one other value for b, and make the same measurements. Based on these experiments, please suggest a formula for ω_r that might be valid for all values of b and k.

Modeling the system

Basic process of modeling a system:

(1) *Draw a diagram of the system.*

(2) *Identify and give symbols for the parameters of the system.*

(3) *Declare the input signal and the system response.*

(4) *Write down a differential equation relating the input signal and the system response, using Newton's "$F = ma$" in the mechanical case or Kirkhoff's laws in the electrical case.*

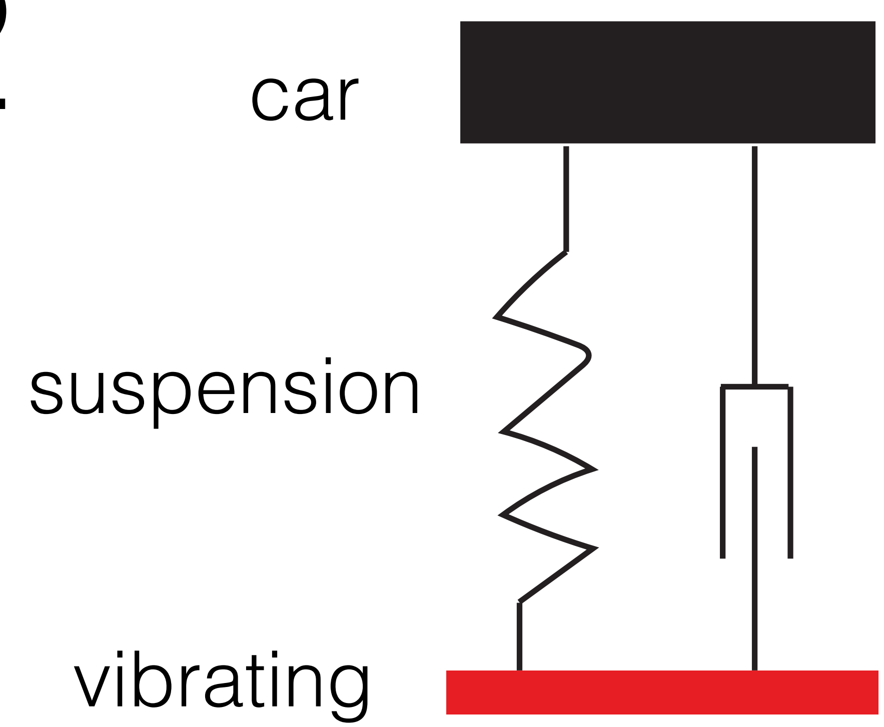
(5) *Rewrite the equation in standard form.*

Standard form:
"Stuff containing output x(t), x'(t) etc. = everything else: input"

Example from AP2II: see board

Activity 2

Car driving on a bumpy road:



(1) *Draw a diagram of the system.*

(2) *Identify and give symbols for the parameters of the system.*

(3) *Declare the input signal and the system response.*

(4) *Write down a differential equation relating the input signal and the system response, using Newton's "$F = ma$" in the mechanical case or Kirkhoff's laws in the electrical case.*

(5) *Rewrite the equation in standard form.*

check with Mathlet Amplitude&Phase 2nd order, III

Review of 18.03

- Operator notation, characteristic polynomial
- Sinusoidals
- Complex exponential
- Solution of homogeneous equations
- Exponential response formula (ERF)

Activity 3

Find the general solution of the equation

$$\ddot{x} + 2\dot{x} + 2x = \cos(3t)\,.$$

A complex replacement is

$$\ddot{z} + 2\dot{z} + 2z = e^{3it}\,.$$

$$x = \frac{1}{85}(-7\cos(3t) + 6\sin(3t)) + ae^{-t}\cos t + be^{-t}\sin t\,.$$

Gain

Consider more general case: $P(D)x = Q(D)(\text{a sinusoid})$

Using complex replacement: $P(D)x = Q(D)e^{i\omega t}$.

Assuming (again) solutions of the form $z_p = G(\omega)e^{i\omega t}$ and plugging in we find (LHS):

$$P(D)z_p = P(D)G(\omega)e^{i\omega t} = G(\omega)P(i\omega)e^{i\omega t}$$

For the RHS, we have $Q(D)e^{i\omega t} = Q(i\omega)e^{i\omega t}$, thus

$$G(\omega)P(i\omega)e^{i\omega t} = Q(i\omega)e^{i\omega t}$$

$$G(\omega) = \frac{Q(i\omega)}{P(i\omega)}$$

complex gain

Writing G in polar form, $G = |G|e^{-i\phi}$ we get

$$z_p = Ge^{i\omega t} = |G|e^{i(\omega t - \phi)}$$

$$x_p = Re(z_p) = |G|\cos(\omega t - \phi)$$

gain, complex gain & phase lag

$$G(\omega) = \frac{Q(i\omega)}{P(i\omega)}$$

complex gain

$$x_p = Re(z_p) = |G| \cos(\omega t - \phi)$$

gain

phase lag

$$g(\omega) = |G(\omega)| \quad , \quad \phi(\omega) = -\arg(G(\omega))$$