18.369 Problem Set 5

Due Monday, 4 April 2016

Problem 1: Group Velocity and Material Dispersion

In class, we showed (following the book) that the group velocity $d\omega/dk = \langle H_k, \frac{\partial \hat{\Theta}_k}{\partial k} H_k \rangle / \langle H_k, H_k \rangle$ was equal to Poynting flux divided by energy density (both averaged over the unit cell).

Now, go through the same derivation, but in this case assume that we have a lossless *dispersive* material with a real $\varepsilon(\mathbf{x}, \omega)$. In this case, when you take the k derivative, apply the chain rule to obtain a term with $\frac{\partial \varepsilon}{\partial \omega} \frac{d\omega}{dk}$ on the right-hand side. Solve for $d\omega/dk$ and show that it is Poynting flux divided by energy density, but the energy density is now the "Brillouin" energy density of a lossless dispersive medium, which we gave in the notes for Lecture 6:

$$\frac{1}{4}\left[\frac{\partial(\omega\varepsilon)}{\partial\omega}|\mathbf{E}|^2 + |\mathbf{H}|^2\right]$$

(for $\mu = 1$, where we have an additional $1/2$ factor from the time-average).

Problem 2: Projected Band Diagrams and Fabry-Perot Waveguides

(a) Starting with the bandgap1d.ctl MPB control file from problem set 4, which computes the frequencies as a function of k_x. Modify it to compute the frequencies as a function of k_y for some range of k_y (e.g. 0 to 2, in units of $2\pi/a$ ... recall that the k_y Brillouin zone is infinite!) for some fixed value of k_x, and to use $\varepsilon_2 = 2.25$ instead of 1.[1]

Compute and plot the TM projected band diagram for the quarter-wave stack with ε of 12 and 2.25. That is, plot $\omega(k_y)$ for several bands, first with $k_x = 0$, then $k_x = 0.1$, then 0.2, then ... then 0.5, and interpolate intermediate k_x to shade in the "continuum" regions of the projected bands. Verify that the extrema of these continua lie at either $k_x = 0$ or $k_x = 0.5$ (in units of $2\pi/a$), i.e. at the B.Z. edges.

(b) Modify the MPB defect1d.ctl file from problem set 4 to compute the defect mode as a function of k_y (for $k_x = 0$).

Changing a single ε_2 layer by $\Delta\varepsilon = 4$, with an $N = 20$ supercell, plot the first 80 bands as a function of k_y for some reasonable range of k_y. Overlay your TM projected band diagram from part (a), above, to show that the bands fall into two categories: modes that fall within the projected "continuum" regions from part (a), and discrete guided bands that lie within the empty spaces. (If there are any bands *just* outside the *edge* of the continuum region, you may need to increase the supercell size to check whether those bands are an artifact of the finite size.) Plot the fields for the guided bands (a couple of nonzero k_y points will do) to show that they are indeed localized.

[1] You might want to add a "kx" parameter via "(define-param kx 0)" so that you can change k_x from the command line with "mpb kx=0.3 ...".