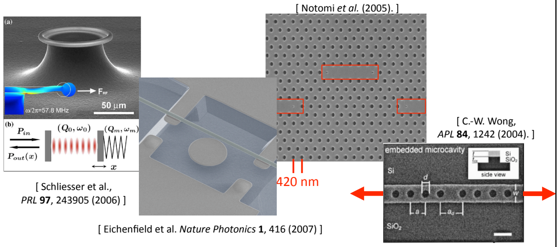
Resonance

an oscillating mode trapped for a long time in some volume
(of light, sound, …)
frequency ω_0
lifetime $\tau \gg 2\pi/\omega_0$
quality factor $Q = \omega_0\tau/2$
energy ~ $e^{-\omega_0 t/Q}$
modal volume V



[Notomi *et al.* (2005).]
[C.-W. Wong, *APL* **84**, 1242 (2004).]
[Schliesser et al., *PRL* **97**, 243905 (2006)]
[Eichenfield et al. *Nature Photonics* **1**, 416 (2007)]

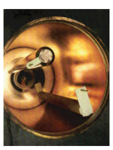
Why Resonance?

an oscillating mode trapped for a long time in some volume

- long time = narrow bandwidth … filters (WDM, etc.)
  — $1/Q$ = fractional bandwidth
- resonant processes allow one to "impedance match" hard-to-couple inputs/outputs
- long time, small V … enhanced wave/matter interaction
  — lasers, nonlinear optics, opto-mechanical coupling, sensors, LEDs, thermal sources, …

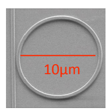
How Resonance?

need mechanism to trap light for long time



[llnl.gov]

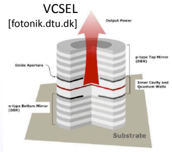
metallic cavities:
good for microwave, dissipative for infrared

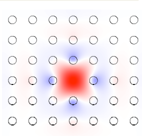


[Xu & Lipson (2005)]

ring/disc/sphere resonators:
a waveguide bent in circle, bending loss ~ exp(−radius)

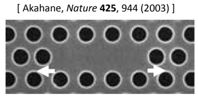
VCSEL
[fotonik.dtu.dk]





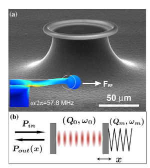
photonic bandgaps
(complete or partial + index-guiding)

[Akahane, *Nature* **425**, 944 (2003)]



(planar Si slab)

Understanding Resonant Systems



[Schliesser et al., *PRL* **97**, 243905 (2006)]

- Option 1: Simulate the whole thing exactly
  — many powerful numerical tools
  — limited insight into a single system
  — can be difficult, especially for weak effects (nonlinearities, etc.)
- Option 2: Solve each component separately, couple with explicit perturbative method (one kind of "coupled-mode" theory)
- Option 3: abstract the geometry into its most generic form
  …write down the *most general* possible equations
  …constrain by fundamental laws (conservation of energy)
  …solve for universal behaviors of a whole class of devices
  … characterized via specific parameters from option 2

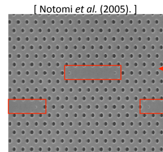
"Temporal coupled-mode theory"

- Generic form developed by Haus, Louisell, & others in 1960s & early 1970s
  – Haus, *Waves & Fields in Optoelectronics* (1984)
  – Reviewed in our *Photonic Crystals: Molding the Flow of Light*, 2nd ed., ab-initio.mit.edu/book
- Equations are generic ⇒ reappear in many forms in many systems, rederived in many ways (e.g. Breit–Wigner scattering theory)
  – full generality is not always apparent

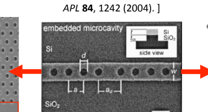
(modern name coined by S. Fan @ Stanford)

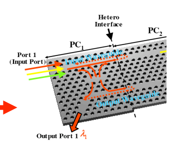
TCMT example: a linear filter

[Notomi *et al.* (2005).]

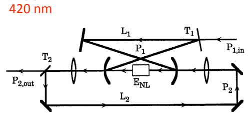


[C.-W. Wong, *APL* **84**, 1242 (2004).]



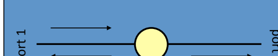


[Takano *et al.* (2006)]



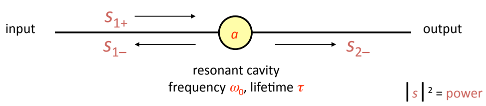
[Ou & Kimble (1993)]

= *abstractly:*
two single-mode i/o ports
+ one resonance



resonant cavity
frequency ω_0, lifetime τ

Temporal Coupled-Mode Theory
for a linear filter



$|s|^2$ = power

$|a|^2$ = energy

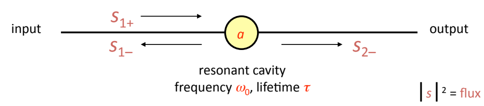
$$\frac{da}{dt} = -i\omega_0 a - \frac{2}{\tau}a + \sqrt{\frac{2}{\tau}}s_{1+}$$

$$s_{1-} = -s_{1+} + \sqrt{\frac{2}{\tau}}a, \quad s_{2-} = \sqrt{\frac{2}{\tau}}a$$

assumes only:
- exponential decay (strong confinement)
- linearity
- conservation of energy
- time-reversal symmetry

can be relaxed

Temporal Coupled-Mode Theory
for a linear filter

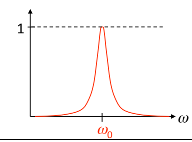


$|s|^2$ = flux

$|a|^2$ = energy

transmission T

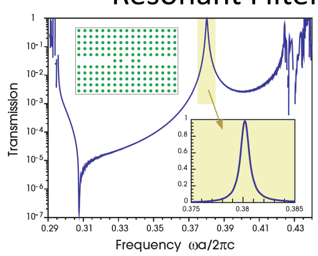
$= |s_{2-}|^2 / |s_{1+}|^2$



T = Lorentzian filter

$$= \frac{\frac{4}{\tau^2}}{(\omega-\omega_0)^2 + \frac{4}{\tau^2}}$$

Resonant Filter Example

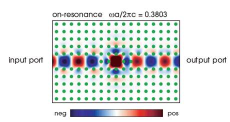


Lorentzian peak, as predicted.

An apparent miracle:

~ 100% transmission at the resonant frequency

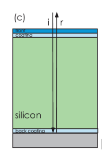
cavity decays to input/output with *equal rates*
⇒ At resonance, reflected wave destructively interferes with backwards-decay from cavity & the two *exactly cancel*.



Some interesting resonant transmission processes

Wireless resonant power transfer
[M. Soljacic, MIT (2007)]
witricity.com

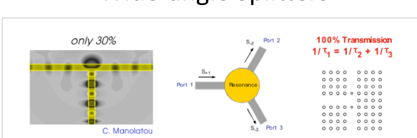
Resonant LED emission
luminus.com



(narrow-band) resonant absorption in a thin-film photovoltaic

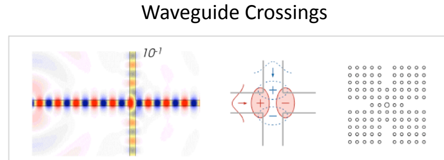
[e.g. Ghebrebrhan (2009)]

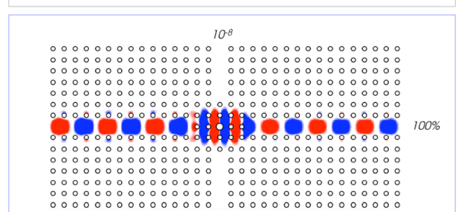
Wide-angle Splitters



[S. Fan *et al.*, *J. Opt. Soc. Am. B* **18**, 162 (2001)]

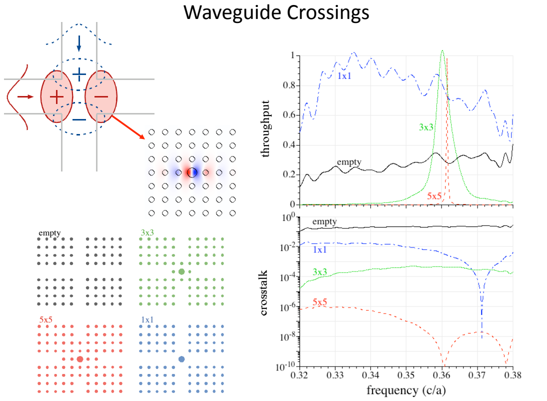
Waveguide Crossings



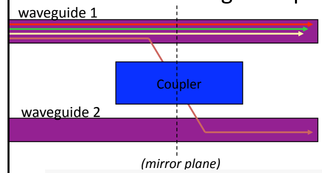


[S. G. Johnson *et al.*, *Opt. Lett.* **23**, 1855 (1998)]

Waveguide Crossings

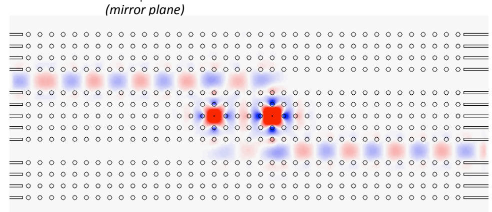


Another interesting example: Channel-Drop Filters



Perfect channel-dropping if:

Two resonant modes with:
- even and odd symmetry
- equal frequency (degenerate)
- equal decay rates



[S. Fan *et al.*, *Phys. Rev. Lett.* **80**, 960 (1998)]

Dimensionless Losses: *Q*

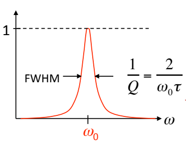
$Q = \omega_0 \tau / 2$

quality factor Q = # optical periods for energy to decay by exp(−2π)

energy ~ $\exp(-\omega_0 t/Q) = \exp(-2t/\tau)$

in frequency domain: $1/Q$ = bandwidth

from temporal coupled-mode theory:



$\frac{1}{Q} = \frac{2}{\omega_0 \tau}$

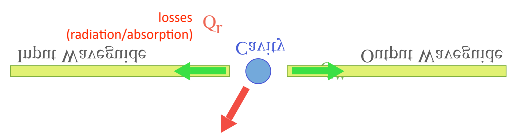
T = Lorentzian filter

$= \frac{\frac{4}{\tau^2}}{(\omega - \omega_0)^2 + \frac{4}{\tau^2}}$

...quality factor Q

More than one *Q*...

A simple model device (filters, bends, ...):



$\frac{1}{Q} = \frac{1}{Q_r} + \frac{1}{Q_w}$

Q = lifetime/period
= frequency/bandwidth

We want: $Q_r >> Q_w$

TCMT ⇒

1 – transmission ~ $2Q / Q_r$

***worst case*:** high-Q (narrow-band) cavities

Nonlinearities + Microcavities?

weak effects
Δn < 1%

very intense fields
& sensitive to small changes

A simple idea:
for the same input power, nonlinear effects are stronger in a microcavity

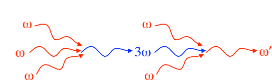
That's not all!
nonlinearities + microcavities
= *qualitatively* new phenomena

Nonlinear Optics

Kerr nonlinearities $\chi^{(3)}$: *(polarization ~ E^3)*

- Self-Phase Modulation (SPM)
  = change in refractive index(ω) ~ |**E**(ω)|²
- Cross-Phase Modulation (XPM)
  = change in refractive index(ω) ~ |**E**(ω_2)|²

- Third-Harmonic Generation (THG) & down-conversion (FWM)
  = ω → 3ω, and back
- etc...



Second-order nonlinearities $\chi^{(2)}$: *(polarization ~ E^2)*

- Second-Harmonic Generation (SHG) & down-conversion
  = ω → 2ω, and back
- Difference-Frequency Generation (DFG) = $\omega_1, \omega_2 \rightarrow \omega_1 - \omega_2$
- etc...

Nonlinearities + Microcavities?

weak effects
$\Delta n < 1\%$

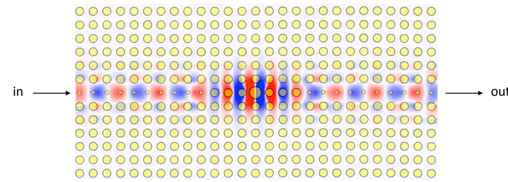
very intense fields
& sensitive to small changes

A simple idea:
for the same input power, nonlinear effects are stronger in a microcavity

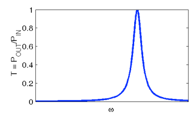
That's not all!
nonlinearities + microcavities
= *qualitatively* new phenomena

let's start with a well-known example from 1970's…

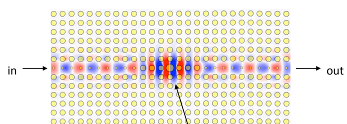
A Simple Linear Filter



Linear response:
Lorenzian Transmisson

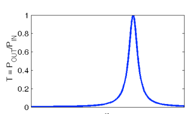


Filter + Kerr Nonlinearity?



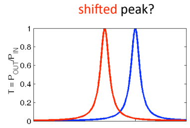
Kerr nonlinearity:
$\Delta n \sim |\mathbf{E}|^2$

Linear response:
Lorenzian Transmisson



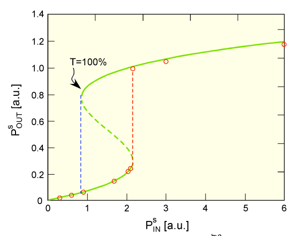
+ nonlinear index shift = ω shift

shifted peak?

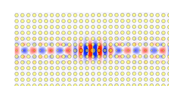


Optical Bistability

[Felber and Marburger., *Appl. Phys. Lett.* **28**, 731 (1978).]



Logic gates, switching, rectifiers, amplifiers, isolators, …



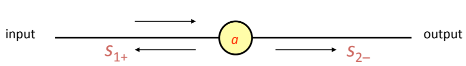
[Soljacic *et al.*, *PRE Rapid. Comm.* **66**, 055601 (2002).]

Bistable (hysteresis) response
(& even multistable for multimode cavity)

Power threshold ~ V/Q^2
(in cavity with $V \sim (\lambda/2)^3$,
for Si and telecom bandwidth power ~ mW)

TCMT for Bistability

[Soljacic *et al.*, *PRE Rapid. Comm.* **66**, 055601 (2002).]



resonant cavity
frequency ω_0, lifetime τ,
SPM coefficient $\alpha \sim \chi^{(3)}$
(from perturbation theory)

$|s|^2$ = power
$|a|^2$ = energy

$$\frac{da}{dt} = -i(\omega_0 - \alpha|a|^2)a - \frac{2}{\tau}a + \sqrt{\frac{2}{\tau}}s_{1+}$$

$$s_{1-} = -s_{1+} + \sqrt{\frac{2}{\tau}}a, \quad s_{2-} = \sqrt{\frac{2}{\tau}}a$$

gives cubic equation for transmission
… bistable curve

TCMT + Perturbation Theory

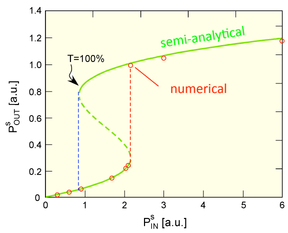
SPM = small change in refractive index
… evaluate $\Delta\omega$ by 1st-order perturbation theory

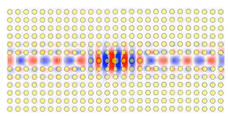
$$\alpha_{ii} = \frac{1}{8}\frac{\int d^3\mathbf{x}\ \varepsilon\chi^{(3)}|\mathbf{E}_i\cdot\mathbf{E}_i|^2 + |\mathbf{E}_i\cdot\mathbf{E}_i^*|^2}{\left[\int d^3\mathbf{x}\ \varepsilon\,|\mathbf{E}_i|^2\right]^2}$$

⇒ all relevant parameters (ω, τ or Q, α) can be computed from the resonant mode of the linear system

Accuracy of Coupled-Mode Theory

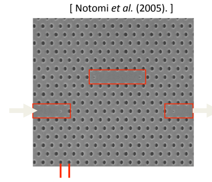
[Soljacic *et al.*, *PRE Rapid. Comm.* **66**, 055601 (2002).]





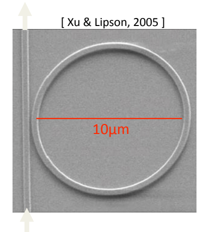
Optical Bistability in Practice

[Notomi *et al.* (2005).]



Q ~ 30,000
V ~ 10 optimum
Power threshold ~ 40 µW

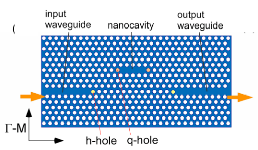
[Xu & Lipson, 2005]

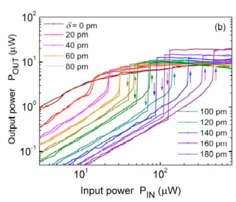


Q ~ 10,000
V ~ 300 optimum
Power threshold ~ 10 mW

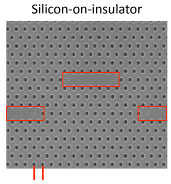
Experimental Bistable Switch

[Notomi *et al.*, *Opt. Express* **13** (7), 2678 (2005).]





Silicon-on-insulator



Q ~ 30,000
Power threshold ~ 40 µW
Switching energy ~ 4 pJ