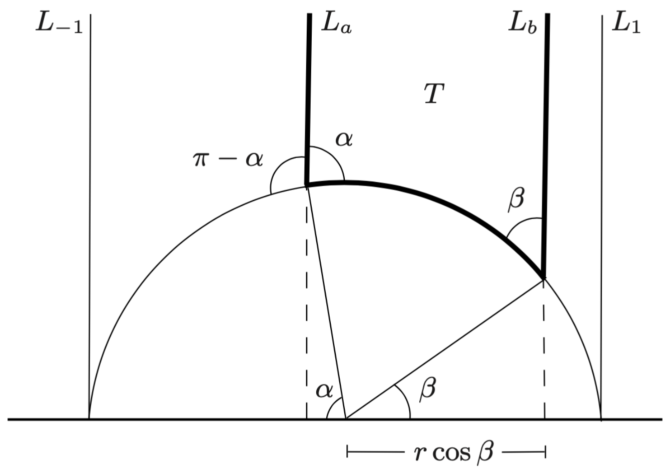
BELYI MAPS AND DESSINS D'ENFANTS
HOMEWORK #6

Exercise 6.1. Let T be a hyperbolic triangle with one vertex at infinity, and the other two vertices on the unit circle. Denote the angles of T at these latter two vertices by α and β, as shown below. Compute the hyperbolic area of T. (Recall that the hyperbolic area is given by

$$a(T) = \int_T \frac{dxdy}{y^2}.$$

Use polar coordinates to compute this integral.)



Exercise 6.2. Let G be a transitive permutation group on a finite set A. Recall the following definitions.

- A block of G is a nonempty subset B of A such that for all $\sigma \in G$, either $\sigma(B) = B$ or $\sigma(B) \cap B = \varnothing$, where $\sigma(B) = \{\sigma(b) : b \in B\}$.
- G is primitive if the only blocks of G are are the trivial ones: the sets of size 1, and A itself.

(a) If B is a block and $a \in B$, show that the set

$$\mathrm{Stab}_G(B) = \{\sigma \in G \mid \sigma(B) = B\}$$

is a subgroup of G containing $\mathrm{Stab}_G(a)$.

(b) If B is a block and $\sigma_1(B), \sigma_2(B), \ldots, \sigma_n(B)$ are the distinct images of B under elements of G, show that these form a partition of A.

(c) Prove that G is primitive if and only if for each $a \in A$, the only subgroups of G containing $\mathrm{Stab}_G(a)$ are $\mathrm{Stab}_G(a)$ and G itself, i.e., $\mathrm{Stab}_G(a)$ is a *maximal* subgroup of G.

Date: May 10, 2021.