BELYI MAPS AND DESSINS D'ENFANTS
HOMEWORK #5

Exercise 5.1. Let X and Y be topological spaces. Suppose G is a group acting (continuously) on a topological space X, and let $\pi : X \to G\backslash X$ be the quotient map taking $x \mapsto Gx$.

(a) Show that a map $f : G\backslash X \to Y$ is continuous if and only if $f \circ \pi : X \to Y$ is continuous and G-invariant, i.e., $(f \circ \pi)(gx) = (f \circ \pi)(x)$ for all $g \in G$ and $x \in X$.

(b) Show that there is a bijective correspondence

$$\left\{ \begin{array}{c} \text{continuous maps} \\ f : G\backslash X \to Y \end{array} \right\} \quad \overset{\sim}{\longleftrightarrow} \quad \left\{ \begin{array}{c} G\text{-invariant continuous} \\ \text{maps } h : X \to Y \end{array} \right\}.$$

Exercise 5.2. Let $\zeta = e^{2\pi i/r}$ be a primitive r^{th} root of unity. Define $\sigma, \tau : \widehat{\mathbb{C}} \to \widehat{\mathbb{C}}$ by $z \mapsto \zeta z$ and $z \mapsto 1/z$, respectively.

(a) Show that the subgroup $G := \langle \sigma, \tau \rangle$ of $\mathrm{Aut}(\widehat{\mathbb{C}})$ is isomorphic to D_{2r}, the dihedral group of order $2r$.

(b) Let $\pi : \widehat{\mathbb{C}} \to G\backslash\widehat{\mathbb{C}}$ be the quotient map. Show that π has 3 ramification values whose corresponding ramification points have ramification indices $2, 2, r$, respectively.

Date: April 1, 2021.