BELYI MAPS AND DESSINS D'ENFANTS
HOMEWORK #4

Exercise 4.1. Use the fact that $\chi(\mathbb{S}^2) = 2$ to show that there are only five regular polyhedra. [*Hint*: Consider subdivisions of the sphere into n-gons such that exactly m edges meet at each vertex, $m, n \geq 3$. Show that $nf = 2e = mv$, so that

$$\frac{1}{e} + \frac{1}{2} = \frac{1}{m} + \frac{1}{n}$$

and consider the solutions with $n = 3, 4, 5$ and $n \geq 6$ in turn.]

Exercise 4.2. Let $E : Y^2Z = F(X, Z)$ be an elliptic curve given in Weierstrass form. Let $x = X/Z$ and $y = Y/Z$ be affine coordinates on the affine open U where $Z \neq 0$ and let $\omega = \frac{dx}{y}$. We showed in class that ω is holomorphic on U. Let $\infty = [0 : 1 : 0]$.

(a) Show that $\mathrm{ord}_\infty(x) = -2$ and $\mathrm{ord}_\infty(y) = -3$. [*Hint*: What are the degrees of x and y? How many points in their respective fibers above $\infty = [1 : 0] \in \mathbb{P}^1$?]

(b) Show that ω is also holomorphic at $[0 : 1 : 0]$ and conclude that it is a holomorphic differential on all of E. [*Hint*: Compute $\mathrm{ord}_\infty(\omega)$ using the fact that $\mathrm{ord}_\infty(dx) = \mathrm{ord}_\infty(x) - 1$.]

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