18.700 MIDTERM EXAM 2 REVIEW

1. Overview of Topics

(1) Invertibility and Isomorphisms
  (a) Linear maps as matrix multiplication
    (i) ${}_{\mathcal{C}}[T]_{\mathcal{B}}[v]_{\mathcal{B}} = [T(v)]_{\mathcal{C}}$
  (b) Change of basis
    (i) $[T]_{\mathcal{C}} = ({}_{\mathcal{B}}[I]_{\mathcal{C}})^{-1}\,[T]_{\mathcal{B}}\;{}_{\mathcal{B}}[I]_{\mathcal{C}}$
(2) Eigenvalues and Eigenvectors
  (a) Invariant subspaces
  (b) Definition of eigenvalue and eigenvector
  (c) Characterization of eigenvalues in terms of $T - \lambda I$
(3) The Minimal Polynomial
  (a) Definition of the minimal polynomial of a linear operator
  (b) Roots of the minpoly(T) are exactly the eigenvalues of T
  (c) Invertibility and the minimal polynomial
(4) Upper Triangular Matrices
  (a) Characterizing when $[T]_{\mathcal{B}}$ is upper triangular in terms of invariant subspaces
  (b) Eigenvalues of an upper triangular matrix
  (c) Upper triangularizability in terms of the minimal polynomial
  (d) Over $\mathbb{C}$, for every linear operator T, there is a basis $\mathcal{B}$ such that $[T]_{\mathcal{B}}$ is upper triangular
(5) Diagonalizability
  (a) Definition of eigenspace
  (b) Criteria for diagonalizability
  (c) Criterion for diagonalizability in terms of the minimal polynomial
(6) Inner Products and norms
  (a) Definition of an inner product
  (b) Definition of the norm associated to an inner product
  (c) Orthogonality
  (d) Pythagorean Theorem, Cauchy-Schwarz Inequality, Triangle Inequality
(7) Orthonormal Bases
  (a) Orthogonality and linear independence
  (b) Expressing vectors as linear combinations of an orthonormal basis: if $e_1, \dots, e_n$ is an orthonormal basis, then $v = \langle v, e_1\rangle e_1 + \cdots + \langle v, e_n\rangle e_n$ for all $v \in V$
  (c) Gram-Schmidt orthogonalization procedure
  (d) Over $\mathbb{C}$, for every linear operator T, there is an orthonormal basis $\mathcal{E}$ such that $[T]_{\mathcal{E}}$ is upper triangular
(8) Orthogonal Complements and Minimization
  (a) Definition of orthogonal complement
  (b) If U is a finite-dimensional subspace of V, then $V = U \oplus U^{\perp}$

(c) Definition of orthogonal projection onto a subpsace
(d) Distance to a subspace minimized by orthogonal projection

2. Practice Problems

The following list refers to exercises in Axler's *Linear Algebra Done Right*, 4th edition.

(1) 3D #7
(2) 3D #9
(3) 3D #10
(4) 3D #17
(5) 5A #5
(6) 5A #6
(7) 5A #7
(8) 5A #10
(9) 5A #13
(10) 5A #37
(11) 5B #3
(12) 5B #12
(13) 5B #14
(14) 5C #4
(15) 5C #5
(16) 5D #4
(17) 5D #9
(18) 5D #11
(19) 6A #2
(20) 6A #21
(21) 6A #26
(22) 6B #4
(23) 6B #8
(24) 6C #1
(25) 6C #9
(26) 6C #15