18.700 PROBLEM SET 1

Due Thursday, September 12 at 11:59 pm on Canvas

Collaborated with:
Sources used:

Problem 1. (3 points) Do the planes $x_1 + 2x_2 + x_3 = 4$, $x_2 - x_3 = 1$ and $x_1 + 3x_2 = 0$ have at least one common point of intersection?

Problem 2. (3 points) Give an example of a 3×3 matrix A with real entries whose reduced row echelon form is

$$\begin{pmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 1/3 \\ 0 & 0 & 0 \end{pmatrix},$$

and such that every entry of A is a nonzero integer.

Problem 3. (5 points) Fix $a, b, c, d \in \mathbb{F}$ and consider the matrix

$$A := \begin{pmatrix} a & b & 1 & 0 \\ c & d & 0 & 1 \end{pmatrix}.$$

Assuming $a \neq 0$ and $c \neq 0$, compute the reduced row echelon form of A. (*Hint*: You will have to deal with two cases, depending on whether some quantity is zero or not.)

Problem 4. (4 points) For each of the following augmented matrices, determine the value(s) of h such that corresponding linear system is consistent.

(a)

$$\left(\begin{array}{cc|c} 1 & h & -3 \\ -2 & 4 & 6 \end{array}\right)$$

(b)

$$\left(\begin{array}{cc|c} 1 & 3 & -2 \\ -4 & h & 8 \end{array}\right)$$

Problem 5. (3 points) Let

$$S := \left\{ \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} t + \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} : t \in \mathbb{F} \right\}.$$

Give a linear system whose solution set is S.

Problem 6. (6 points)

(a) Suppose that $p(t) = a_0 + a_1 t + a_2 t^2$ is a quadratic polynomial with $a_0, a_1, a_2 \in \mathbb{F}$, whose graph passes through the points $(1, 12), (2, 15)$, and $(3, 16)$. Find the coefficients a_0, a_1, a_2 by solving the following linear system.

$$\begin{array}{llll} a_0 & +a_1(1) & +a_2(1)^2 & = 12 \\ a_0 & +a_1(2) & +a_2(2)^2 & = 15 \\ a_0 & +a_1(3) & +a_2(3)^2 & = 16 \end{array}$$

(b) Suppose that $p(t) = a_0 + a_1 t + \cdots + a_n t^n$ is a polynomial of degree n with $a_0, a_1, \ldots, a_n \in \mathbb{F}$, whose graph passes through the points $(u_1, v_1), (u_2, v_2), \ldots, (u_{n+1}, v_{n+1})$. Determine a linear system that the coefficients $a_0, a_1, \ldots, a_n$ must satisfy, and write down its corresponding augmented matrix.