KERNELS AND IMAGES OF LINEAR MAPS WORKSHEET

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(1) Let V and W be vector spaces and $T : V \to W$ be linear.
   (a) Show that $\ker(T)$ is a subspace of V.

   (b) Show that $\text{img}(T)$ is a subspace of W.

(2) Define $\varphi \in \mathcal{L}(\mathbb{C}^3, \mathbb{C})$ by $\varphi(z_1, z_2, z_3) = z_1 + 2z_2 + 3z_3$. Compute $\ker(\varphi)$ and $\operatorname{img}(\varphi)$.

(3) Define $D \in \mathcal{L}(\mathcal{P}(\mathbb{R}), \mathcal{P}(\mathbb{R}))$ by $D(p) = p'$. Compute $\ker(D)$ and $\operatorname{img}(D)$.