SUMS AND LINEAR INDEPENDENCE WORKSHEET

SEPTEMBER 18, 2024

(1) Let V be a vector space and V_1, V_2 be subspaces. Show that $V_1 + V_2$ is a direct sum iff $V_1 \cap V_2 = \{0\}$.

(2) Let $n \in \mathbb{Z}_{>0}$ be a positive integer and let $V := \mathbb{F}^n$. Let

$$e_1 := (1, 0, \ldots, 0), e_2 := (0, 1, 0, \ldots, 0), \ldots, e_n := (0, \ldots, 0, 1),$$

so e_k is the n-tuple with a 1 in the k^{th} position and zeroes elsewhere.

(a) Show that $e_1, \ldots, e_n$ spans $\mathbb{F}^n$.

(b) Show that $e_1, \ldots, e_n$ is linearly independent.

(3) Let

$$v_1 := (1, 3, -4, 2), \qquad v_2 := (2, 2, -4, 0),$$
$$v_3 := (1, -3, 2, -4), \qquad v_4 := (-1, 0, 1, 0)\,.$$

(a) Show that $v_1, \dots, v_4$ are linearly dependent.

(b) Find the smallest k such that $v_k \in \operatorname{span}(v_1, \dots, v_{k-1})$, and write v_k as a linear combination of $v_1, \dots, v_{k-1}$.