SINGULAR VALUE DECOMPOSITION WORKSHEET

NOVEMBER 20, 2024

1. Compute an SVD of $A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{pmatrix}$.

2. Use your answer to the previous question to find an SVD for A^T *without* having to recompute the decomposition from scratch.

3. (a) Show that if $\mathbf{v}$ is an eigenvector of A^TA and $A\mathbf{v} \neq \mathbf{0}$, then $A\mathbf{v}$ is an eigenvector of AA^T with the same eigenvalue.

   (b) Show that if $\mathbf{u}$ is an eigenvector of AA^T and $A^T\mathbf{u} \neq \mathbf{0}$, then $A^T\mathbf{u}$ is an eigenvector of A^TA with the same eigenvalue.

   (c) Conclude that A^TA and AA^T have the same nonzero eigenvalues.