DIAGONALIZATION WORKSHEET

OCTOBER 23, 2024

(1) Define

$$T : \mathbb{F}^3 \to \mathbb{F}^3$$
$$(x, y, z) \mapsto (4x + 5y - 10z, -y + 14z, 6z).$$

(a) Compute $[T]_{\mathcal{E}}$ where $\mathcal{E}$ is the standard basis of $\mathbb{F}^3$.

(b) Let $A := [T]_{\mathcal{E}}$. Determine the eigenvalues of T. (Note that A is upper triangular.)

(c) Compute the eigenvectors of T.

(d) Let $\mathcal{B}$ be the basis of $\mathbb{F}^3$ consisting of the eigenvectors you computed in the previous part. Compute $D := [T]_{\mathcal{B}}$, $P := {}_{\mathcal{E}}[I]_{\mathcal{B}}$, and P^{-1}. (To compute P^{-1}, form the augmented matrix $(P \mid I)$ and row reduce.)

(e) Express A in terms of D and P.

(f) Using the previous part, compute A^2 and A^{100}. (*Hint*: Use the previous part!)