EIGENVECTORS AND EIGENVALUES WORKSHEET

OCTOBER 4, 2024

(1) Let D be the diagonal matrix

$$\begin{pmatrix} d_1 & & & \\ & d_2 & & \\ & & \ddots & \\ & & & d_n \end{pmatrix}$$

and define $T : \mathbb{F}^n \to \mathbb{F}^n$ by $T(v) = Dv$. What are the eigenvectors of T? What are their corresponding eigenvalues?

(2) Let
$$T : \mathbb{F}^2 \to \mathbb{F}^2$$
$$\begin{pmatrix} w \\ z \end{pmatrix} \mapsto \begin{pmatrix} -z \\ w \end{pmatrix}.$$

(a) Compute $T \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $T \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Describe T geometrically.

(b) What are the eigenvalues of T when $\mathbb{F} = \mathbb{R}$? When $\mathbb{F} = \mathbb{C}$? (*Hint*: Suppose $T \begin{pmatrix} w \\ z \end{pmatrix} = \lambda \begin{pmatrix} w \\ z \end{pmatrix}$ for some $\lambda, w, z \in \mathbb{F}$. Can you find constraints on w and z?)