Read Sections 9.1, 9.2, 10.2, 10.4 through Theorem 10.33.  
Skip the section on Primality (Theorem 10.6 thru Theorem 10.9) and the proof of Lemma 10.30.

1. Let \( EQ_{BP} = \{ \langle B_1, B_2 \rangle | B_1 \text{ and } B_2 \text{ are equivalent branching programs} \} \).
   Show that \( EQ_{BP} \) is coNP-complete.

2. Recall that \( E_{CFG} = \{ \langle G \rangle | G \text{ is a } CFG \text{ and } L(G) = \emptyset \} \).
   (a) Show that \( E_{CFG} \in P \).
   (b) It is not known whether \( E_{CFG} \in L \). Give evidence to suggest \( E_{CFG} \notin L \) by showing that, if \( E_{CFG} \in L \) then \( L = NL \).

3. Define a two-headed finite automaton (2FA) to be a deterministic finite automaton which has two read-only, bidirectional heads that start at the left end of the input tape and which can be independently controlled to move in either direction. (Assume that the input is given with left and right delimiters around it, and that the machine rejects if it tries to move either head past the delimiters). A 2FA accepts its input by entering a special accept state. A 2FA rejects its input in one of three ways: by entering a special reject state, or by attempting to move either input head past the delimiters, or by not halting.
   (a) Let \( A_{2FA} = \{ \langle T, w \rangle | T \text{ is a 2FA that accepts } w \} \). Prove that \( A_{2FA} \) is in P.
   (b) Prove that P contains a language which is not recognizable by a 2FA.

4. (a) Let \( B = \{ \langle G \rangle | G \text{ is an undirected graph that has } \textbf{at least} \text{ two distinct Hamiltonian paths} \} \). Show that \( B \in D^{SAT} \).
   (b) Let \( C = \{ \langle G \rangle | G \text{ is an undirected graph that has } \textbf{exactly} \text{ two distinct Hamiltonian paths} \} \). Show that \( C \in D^{SAT} \).

5. (a) Describe a \( SAT \)-oracle polynomial-time TM that takes as input an undirected graph \( H \), and outputs the \textbf{size} (number of nodes) of a largest clique in \( H \).
   (b) Describe another \( SAT \)-oracle polynomial-time TM that takes as input an undirected graph \( H \), and outputs the \textbf{list of the nodes} in one of the cliques in \( H \) of the largest size.

6. The class RP is a subset of BPP, where the probabilistic polynomial time decider never accepts for inputs outside the language, thereby exhibiting \textit{one-sided error}. More formally, RP is the collection of languages \( A \) for which a probabilistic polynomial time decider, accepts with probability at least \( \frac{2}{3} \) (or equivalently \( \frac{1}{2} \)) for inputs in \( A \) and accepts with probability 0 for inputs not in \( A \). For example, our proof that \( EQ_{ROBP} \in BPP \) actually shows that \( EQ_{ROBP} \in \text{coRP} \). Prove that if \( NP \subseteq BPP \) then \( NP = RP \).
   (Hint: An RP machine should accept only when it is certain that its input is in the language. How can we be certain that a formula \( \phi \) is satisfiable?)

7* (Optional) A \textbf{two-way PDA} is similar to an ordinary PDA but with a bidirectional input tape head. It is nondeterministic. It accepts by entering a specific accept state \( q_{\text{accept}} \) at any point. Show that any language recognized by an two-way PDA is also in P. (Hint: Section 10.3 in the text may be helpful. Note that you are not otherwise responsible knowing Section 10.3.)