Read Sections 9.1, 9.2, 10.2, 10.4 through Theoreom 10.33.
Skip the section on Primality (Theorem 10.6 thru Theorem 10.9) and the proof of Lemma 10.30.

1. Recall that $E_{CFG} = \{ (G) \mid G$ is a CFG and $L(G) = \emptyset \}$. Say that a problem is $\text{NL-hard}$ if all problems in NL are log-space reducible to it, even though it may not be in NL itself. (Similarly define NP-hard and PSPACE-hard.) It is not known whether $E_{CFG} \in \text{NL}$.

Show that $E_{CFG}$ is NL-hard.

2. Let $\text{ICFL}$ be the class of languages that can be expressed as the intersection of two context-free languages. In other words, $\text{ICFL} = \{ A \mid A = B \cap C$ for some CFLs $B$ and $C \}$.

(a) Prove $\text{ICFL} \subseteq \text{P}$.

(b) Prove that P contains some language which is not in $\text{ICFL}$.

(Hint: a theorem we proved in lecture is useful here.)

3. Let $\text{SAT}_{\geq 2} = \{ \langle \phi \rangle \mid \phi$ is a Boolean formula that has at least two satisfying assignments $\}$.

Let $\text{SAT}_{= 2} = \{ \langle \phi \rangle \mid \phi$ is a Boolean formula that has exactly two satisfying assignments $\}$.

(a) Show that $\text{SAT}_{\geq 2} \in \text{P}^{\text{SAT}}$.

(b) Show that $\text{SAT}_{= 2} \in \text{P}^{\text{SAT}}$.

4. Describe a deterministic polynomial-time oracle TM $M$ which has access to an oracle for $\text{SAT}$, where $M$ takes as input a Boolean formula $\phi$ and performs as follows:

If $\phi$ is satisfiable, $M$ outputs a satisfying assignment of $\phi$.

If $\phi$ is not satisfiable, $M$ outputs unsatisfiable.

5. For branching program $B$ and $w = w_1 \ldots w_m$, where each $w_i \in \{0, 1\}$, let $B(w)$ be the output of $B$ when its input variables $x_1, \ldots, x_m$ are set $x_i = w_i$ for each $i$.

(a) Let $\text{ALL}_{\text{ROBP}} = \{ \langle B \rangle \mid B$ is a read-once branching program and $B(w) = 1$ on all $w \}$. Show that $\text{ALL}_{\text{ROBP}} \in \text{P}$.

(b) Let $\text{ALL}_{\text{BP}} = \{ \langle B \rangle \mid B$ is a branching program and $B(w) = 1$ on all $w \}$. Show that $\text{ALL}_{\text{BP}}$ is coNP-complete.

6. Show that BPP is closed under concatenation.

In other words, show that if $A, B \in \text{BPP}$ then $(A \circ B) \in \text{BPP}$.

7* (Optional) Read the proof of Theorem 9.20.

Prove that an oracle $C$ exists for which $\text{NP}^C \neq \text{coNP}^C$.

The Final Exam will be held Tuesday, December 20, 2022, 1:30 – 4:30, Johnson Track.
It covers Chapters 1, 2 (except 2.4), 3, 4, 5, 6.1, 7, 8, 9.1, 9.2, 10.2 (except the part on Primality), and 10.4 through Theorem 10.33. Emphasis on the 2nd half of the course (Chapters 7 and after).