Read all of Chapter 8.

1. In the **disjoint sets problem**, you are given a collection of sets $S_1, \ldots, S_k$ and an integer $t$, and you determine whether that collection contains $t$ sets that are disjoint from each other. Formulated as a language, let $DS = \{\langle S_1, \ldots, S_k, t \rangle | \text{some } t \text{ of the } S_i \text{ are pairwise disjoint} \}$. Prove that $DS$ is NP-complete.

2. You are given a box and a collection of cards as indicated in the following figure.

Because of the pegs in the box and the notches in the cards, each card will fit in the box in either of two ways. Each card contains two columns of holes, some of which may not be punched out. The goal is to place all of the cards in the box so that they completely cover the bottom of the box, (i.e., so that every hole position is blocked by at least one card that has no hole there). Say that a collection of cards is **solvable** if the goal can be achieved. Let $CARDS = \{\langle c_1, \ldots, c_k \rangle | \text{each } c_i \text{ represents a card and this collection of cards is solvable} \}$. Prove that $CARDS$ is NP-complete.

3. The Japanese game *go-moku* is played by two players, X and O, on a $19 \times 19$ grid. Players take turns placing markers, and the first player to achieve five of her markers consecutively in a row, column, or diagonal is the winner. Consider this game generalized to an $m \times m$ board. Let $GM = \{\langle B \rangle | B \text{ is a configuration in generalized go-moku, where X has a winning strategy} \}$. By a **configuration** we mean a board with markers placed on it, such as may occur in the middle of a play of the game, together with an indication of which player moves next. Show that $GM \in \text{PSPACE}$.

4. Show that $A_{LBA} = \{\langle B, w \rangle | B \text{ is an LBA that accepts input } w \} \in \text{PSPACE}$-complete.

5. Show that $E_{\text{NFA}} = \{\langle A \rangle | A \text{ is an NFA and } L(A) = \emptyset \} \in \text{NL}$-complete.

6. (a) Let $ADD = \{\langle x, y, z \rangle | x, y, z > 0 \text{ are binary integers and } x + y = z \}$. Show $ADD \in \text{L}$.

   (b) Let $PAL-ADD = \{\langle x, y \rangle | x, y > 0 \text{ are binary integers where } x + y \text{ is an integer whose binary representation is a palindrome} \}$. (Note that the binary representation of the sum is assumed not to have leading zeros. A palindrome is a string that equals its reverse.) Show that $PAL-ADD \in \text{L}$.

7.* (optional) Say that some multitape NTM decides a language $A$ in time $O(n)$. Show that a two-tape NTM can decide $A$ in time $O(n)$. 