Read all of Chapter 5 and Section 6.1.

0.1 Read and solve, but do not turn in: Book, 5.14. [TM left end overrun is undecidable]

1. Consider the problem of determining whether a single-tape Turing machine ever writes a blank symbol over a nonblank symbol during the course of its computation on any input string. Formulate this problem as a language and show that it is undecidable.

2. Let $OVERLAP_{CFG} = \{ \langle G, H \rangle \mid G$ and $H$ are CFGs and $L(G) \cap L(H) \neq \emptyset \}$. Show that $OVERLAP_{CFG}$ is undecidable. (Hint: Use a reduction from $PCP$. Given an instance $P = \{ [t_1 b_1], [t_2 b_2], \ldots, [t_k b_k] \}$ of the Post Correspondence Problem, construct CFGs $G$ and $H$ with the rules $G : T \rightarrow t_1 Ta_1 | \cdots | t_k Ta_k | t_1 a_1 | \cdots | t_k a_k$

\[ H : B \rightarrow b_1 Ba_1 | \cdots | b_k Ba_k | b_1 a_1 | \cdots | b_k a_k \]

where $a_1, \ldots, a_k$ are new terminal symbols. Prove that this reduction works.)

3. Let $A$ be a language.

(a) Show that $A$ is Turing-recognizable iff $A \leq_m A_{TM}$.

(b) Show that $A$ is decidable iff $A \leq_m 0^{*}1^{*}$.

4. Let $ALL_{TM} = \{ \langle T \rangle \mid T$ is a TM and $L(T) = \Sigma^{*} \}$. 

(a) Show that $A_{TM} \leq_m ALL_{TM}$ to prove that $\overline{ALL_{TM}}$ is not T-recognizable.

(b) (harder) Show that $A_{TM} \leq_m \overline{ALL_{TM}}$ to prove that $ALL_{TM}$ is not T-recognizable.

(Slight hint: Think about the solution to Pset 2, #6.)

5. In a two-dimensional finite automaton (2DIM-DFA) the input is an $m \times n$ rectangle, for any $m, n \geq 2$. The squares along the boundary of the rectangle contain the symbol # and the internal squares contain symbols over the input alphabet $\Sigma$. The transition function $\delta : Q \times (\Sigma \cup \{#\}) \rightarrow Q \times \{L, R, U, D\}$ indicates the next state and the new head position (Left, Right, Up, Down). The machine accepts when it enters one of the designated accept states. It rejects if it tries to move off the input rectangle or if it never halts. Two such machines are equivalent if they accept the same rectangles.

(a) Let $A_{2DIM-DFA} = \{ \langle B, r \rangle \mid B$ is a 2DIM-DFA and $B$ accepts rectangle $r$}. Show that $A_{2DIM-DFA}$ is decidable.

(b) Let $EQ_{2DIM-DFA} = \{ \langle B, C \rangle \mid B$ and $C$ are equivalent 2DIM-DFAs}. Show that $EQ_{2DIM-DFA}$ is not decidable.

6. Give a Python program that prints itself out, in the spirit of the recursion theorem. If you don’t know Python, use some other programming language or an approximation of one.

(Hint: Make a function that takes a string input. The substring operation $S[1:j]$ and escaped quotes are useful. A clean solution avoids implementation features such as character codes.)

7* (optional) Let $A$ and $B$ be two disjoint languages. Say that language $C$ separates $A$ and $B$ if $A \subseteq C$ and $B \subseteq C$. Describe two disjoint Turing-recognizable languages that aren’t separable by any decidable language and prove that your solution works.

Midterm exam: Thursday, October 27, 2022, 2:30–4:00, top floor of Walker.
Covers Chapters 1, 2 (except 2.4), 3, 4, 5, and 6.1.
Exam is open book, notes, and postings, including electronic versions, from this year’s class.