Read all of Chapter 5 and Section 6.1.

0.1 Read and solve, but do not turn in: Book, 5.10 and 5.11. [TM uses 2nd tape is undecidable]

1. Consider the problem of determining whether a given TM $M$ accepts some palindrome. Formally, let $\text{PAL}_{TM} = \{ \langle M \rangle \mid M$ is a TM and $w = w^R$ for some $w \in L(M) \}$. Show that $\text{PAL}_{TM}$ is undecidable by giving a reduction from some language that we’ve already shown to be undecidable. (You must give a reduction to get full credit.)

2. Let $A$ be a language.
   (a) Show that $A$ is Turing-recognizable iff $A \leq_m A_{TM}$.
   (b) Show that $A$ is decidable iff $A \leq_m 0^*1^*$.

3. Let $\text{FINITE}_{TM} = \{ \langle T \rangle \mid T$ is a TM and $L(T)$ is a finite language $\}$.
   (a) Show that $A_{TM} \leq_m \text{FINITE}_{TM}$ to prove that $\text{FINITE}_{TM}$ is not T-recognizable.
   (b) (harder) Show that $A_{TM} \leq_m \text{FINITE}_{TM}$ to prove that $\text{FINITE}_{TM}$ is not T-recognizable.

4. Let $\text{AMBIG}_{CFG} = \{ \langle G \rangle \mid G$ is an ambiguous CFG $\}$. Show that $\text{AMBIG}_{CFG}$ is undecidable. (Hint: Use a reduction from PCP. Given an instance $P = \langle t_1 b_1, t_2 b_2, \ldots, t_k b_k \rangle$ of the Post Correspondence Problem, construct a CFG $G$ with the rules

   $S \to T \mid B$

   $T \to t_1 T a_1 \mid \cdots \mid t_k T a_k \mid t_1 a_1 \mid \cdots \mid t_k a_k$

   $B \to b_1 B a_1 \mid \cdots \mid b_k B a_k \mid b_1 a_1 \mid \cdots \mid b_k a_k$

   where $a_1, \ldots, a_k$ are new terminal symbols. Prove that this reduction works.)

5. Define a 2-pass pushdown automaton ($2\text{PPDA}$) to be identical to the standard PDA as we have defined, except that it reads the input twice: the input head makes two, left-to-right, passes over the input. When the $2\text{PPDA}$ reaches the end of the input for the first time, it is reset back to the start of the input without changing its state or stack, except that a special symbol is added to the the top of the stack to inform the $2\text{PPDA}$ that the reset has occurred. Thus the $2\text{PPDA}$ can detect when the second pass begins. When the $2\text{PPDA}$ reaches the end of the input for the second time, the input is accepted if the $2\text{PPDA}$ is in an accept state.
   (a) Show that a $2\text{PPDA}$ can recognize the language $\{ a^m b^m c^m d^m \mid m \geq 0 \}$.
   (b) Show that $\text{E}_{2\text{PPDA}} = \{ \langle P \rangle \mid P$ is a $2\text{PPDA}$ where $L(P) = \emptyset \}$ is undecidable.
   Your solution should be detailed enough to show why it does prove that $\text{E}_{2\text{PPDA}}$ is undecidable, yet it doesn’t prove that $\text{E}_{PDA}$ is undecidable (since $\text{E}_{PDA}$ is decidable).

6. Give a Python program that prints itself out, in the spirit of the recursion theorem. If you don’t know Python, use some other programming language or an approximation of one.
   (Hint: Assign a string to $S$ to serve as a template. The substring operation $S[i:j]$ and escaped quotes are useful. A clean solution avoids implementation features such as character codes.)

7.* (optional) Show that $\text{EQ}_{TM} \not\leq_m \text{EQ}_{TM}$.

**Midterm exam**: Thursday, October 26, 2023, 2:30–4:00, top floor of Walker.
Covers Chapters 1, 2 (except 2.4), 3, 4, 5, and 6.1. The midterm exam is closed book. No access to the textbook, course notes (printed or electronic), or the internet is allowed. You are allowed to bring and use a one page (printed on both sides) summary sheet (also known as a cheat sheet) of your choice to the midterm.