Read all of Chapters 3 and 4.

0.1 Read and solve, but do not turn in: Book, 2.16. [CFLs closed under $\cup$, $\circ$, $\ast$]
Solve by using both CFGs and PDAs.

0.2 Read and solve, but do not turn in: Book, 2.18. [(CFL $\cap$ regular) is a CFL]
Note, problems marked with $\Delta$ have solutions in the book.

0.3 Read and solve, but do not turn in: Book, 2.26. [Chomsky normal form]

0.4 Read and solve, but do not turn in: Book, 2.30c. [CFL Pumping lemma]

1. Let $\Sigma = \{0, 1, \#\}$ and let $B = \{x\#y \mid x, y \in \{0, 1\}^* \text{ and } x \neq y\}$. Show that $B$ is a CFL.

2. Let $\Sigma = \{1, 2, 3, 4\}$.
   (a) Let $C = \{w \mid w$ has equal numbers of $1$s and $2$s, and equal numbers of $3$s and $4$s$\}$.
   Show that $C$ is not context free.
   (b) Use (a) to show that the class of CFLs isn’t closed under complement and intersection.
   (c) Let $D = C \cup (\Sigma\Sigma)^\ast$. Is $D$ a CFL? Prove your answer.
   (d) Let $E = C \cup \Sigma(\Sigma\Sigma)^\ast$. Is $E$ a CFL? Prove your answer.

3. **A Turing machine with left reset** is similar to an ordinary one-tape TM, but the transition function has the form $\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{R, \text{RESET}\}$. If $\delta(q, a) = (r, b, \text{RESET})$, when the machine is in state $q$ reading an $a$, the machine’s head jumps to the left-hand end of the tape after it writes $b$ on the tape and enters state $r$. Note that these machines do not have the usual ability to move the head one symbol left. Show that Turing machines with left reset recognize the class of Turing-recognizable languages.

4. Show that a language is decidable iff some enumerator enumerates the language in string order. (**String order** is the standard length-increasing, lexicographic order, see text p 14).

5. Let $C$ be a language. Prove that $C$ is Turing-recognizable iff a decidable language $D$ exists such that $C = \{x \mid \exists y \in \{0, 1\}^* \ (\langle x, y \rangle \in D)\}$. (Hint: You must prove both directions of the “iff”. The $\leftarrow$ direction is easier. For the $\rightarrow$ direction, think of $y$ as providing additional information that allows you to confirm when $x \in C$, but without the possibility of looping.)

6. Say that a variable $A$ in CFG $G$ is **useless** if $A$ does not appear in any derivation of any string $w \in L(G)$. Given a CFG $G$, consider the problem of testing whether $G$ contains any useless variable(s). Formulate this problem as a language and show that it is decidable.

7* (Optional) Recall the $MS$ operation on languages we defined in Problem Set 1. Is the class of CFLs closed under $MS$? Prove your answer.