Read all of Chapters 3 and 4.

0.1 Read and solve, but do not turn in: Book, 2.16. [CFLs closed under $\cup$, $\circ$, $\ast$]
Solve by using both CFGs and PDAs.

0.2 Read and solve, but do not turn in: Book, 2.18. [(CFL $\cap$ regular) is a CFL]
Note, problems marked with $^A$ have solutions in the book.

0.3 Read and solve, but do not turn in: Book, 2.26. [Chomsky normal form]

0.4 Read and solve, but do not turn in: Book, 2.30c. [CFL Pumping lemma]

1. Let $C = \{zu| z \in 0^* \text{ and } u \in 0^*10^* \text{ where } |u| = |z|\}$. Show that $C$ is a CFL in two ways:
   (a) by giving a CFG that generates $C$, and
   (b) by giving a PDA that recognizes $C$.

2. Let $D = \{tz| z \in 0^* \text{ and } t \in 0^*10^*10^* \text{ where } |t| = |z|\}$.
   (a) Show that $D$ is not a CFL.
   (b) Is $D \cup (\sum\sum)^* \text{ a CFL? Why or why not?}$
   (c) Is $D \cup \sum(\sum\sum)^* \text{ a CFL? Why or why not?}$

3. Say that a variable $A$ in CFG $G$ is usable if it appears in some derivation of some string $w \in L(G)$. Given a CFG $G$ and a variable $A$, consider the problem of testing whether $A$ is usable. Formulate this problem as a language and show that it is decidable.

4. Let a $k$-PDA be a pushdown automaton that has $k$ stacks. Thus a 0-PDA is an NFA and a 1-PDA is a conventional PDA. You already know that 1-PDAs are more powerful (recognize a larger class of languages) than 0-PDAs.
   (a) Give an example to show that 2-PDAs are more powerful than 1-PDAs.
   (b) Show that 3-PDAs are not more powerful than 2-PDAs.
   (Hint: Simulate a Turing machine tape by using two stacks.)

5. Show that a language is decidable iff some enumerator enumerates the language in string order. (String order is the standard length-increasing, lexicographic order, see text p 14).

6. Let $C$ be a language. Prove that $C$ is Turing-recognizable iff a decidable language $D$ exists such that $C = \{x| \exists y \in \{0,1\}^* ((x,y) \in D)\}$. (Hint: You must prove both directions of the “iff”. The (←) direction is easier. For the (→) direction, think of $y$ as providing additional information that allows you to confirm when $x \in C$, but without the possibility of looping.)

7. (Optional) Prove the following stronger form of the pumping lemma. Here both pieces $v$ and $y$ must be nonempty when the string $s$ is broken up. The formal statement follows.

   If $A$ is a context-free language, then there is a number $p$ where, if $s$ is any string in $A$ of length at least $p$, then $s$ may be divided into five pieces, $s = uvxyz$, satisfying the conditions:
   (a) for each $i \geq 0$, $uv^ixy^iz \in A$,
   (b) $v \neq \varepsilon$ and $y \neq \varepsilon$, and
   (c) $|vxy| \leq p$. 