Read all of Chapters 3 and 4.

0.1 Read and solve, but do not turn in: Book, 2.16. \([\text{CFLs closed under } \cup, \circ, *]\)
Solve by using both \( \text{CFGs} \) and \( \text{PDAs} \).

0.2 Read and solve, but do not turn in: Book, 2.18. \([\text{CFL } \cap \text{regular} \text{ is a CFL}]\)
Note, problems marked with \(^\wedge\) have solutions in the book.

0.3 Read and solve, but do not turn in: Book, 2.26. \([\text{Chomsky normal form}]\)

0.4 Read and solve, but do not turn in: Book, 2.30c. \([\text{CFL Pumping lemma}]\)

1. Let \( B = \{a^ib^j \mid i \neq j \text{ and } 2i \neq j\} \). Show that \( B \) is a CFL. (Hint: think about \( B \) differently.)
If you give a \( \text{CFG} \), explain how it works. If you give a \( \text{PDA} \), only describe it informally.

2. Let \( \Sigma = \{0, 1, \#\} \). Let \( C = \{w\#w^R\#w \mid w \in \{0, 1\}^*\} \). \((w^R \text{ is } w \text{ written in reverse})\)
   (a) Prove that \( C \) is not a CFL.
   (b) Use part (a) to show the class of \( \text{CFLs} \) is not closed under intersection or complement.
   (c) Let \( D = C \cup \Sigma(\Sigma\Sigma\Sigma)^* \). Use part (a) to show that \( D \) is not a CFL.

3. A useless state in a pushdown automaton is never entered on any input string on any computation thread. Consider the problem of determining whether a pushdown automaton has any useless states. Formulate this problem as a language and show that it is decidable. (Hint: Use a theorem from lecture to give a short solution.)

4. Upon receiving your deterministic, single-tape Turing machine back from the Turing Machine Repair Service, you find to your dismay that when it tries to write a symbol on any tape cell, it is only able to squirt a blot of ink on that cell, or it may leave the symbol it read unchanged. When it reads a blot, it treats it as a new symbol \( \blot \). Fortunately, you recall hearing somewhere that this does not diminish the class of recognized languages. All you need now is the proof...
   (Hint: Use lots of tape.)

5. Show that a language is decidable iff some enumerator enumerates the language in string order. Make sure that your solution works even when the enumerator’s language is finite. \((\text{String order} \text{ is the standard length-increasing, lexicographic order, see text p } 14)\).

6. Let \( C \) be a language. Prove that \( C \) is Turing-recognizable iff a decidable language \( D \) exists such that \( C = \{x \mid \exists y \in \{0, 1\}^* \langle x, y \rangle \in D\} \). (Hint: You must prove both directions of the \( \iff \) \. The \( \langle \rightarrow \rangle \) direction is easier. For the \( \langle \rightarrow \rangle \) direction, think of \( y \) as providing additional information that allows you to confirm when \( x \in C \), but without the possibility of looping.)

7. \( \star \) (Optional) Let \( M \) be a single tape TM that cannot write on the portion of the tape which contains the input string. Show that \( L(M) \) is a regular language.