Read all of Chapters 1 and 2 except Section 2.4.

0.1 Read and solve, but do not turn in: Book, 1.14. [Swapping NFA accept/non-accept states]

0.2 Read and solve, but do not turn in: Book, 1.31. [Closure under reversal]

0.3 Read and solve, but do not turn in: Book, 1.46b. [Pumping lemma]

0.4 Read and solve, but do not turn in: Book, 2.4 and 2.5. [Practice with CFLs]

You can assume the results from the above problems when solving the problems below.

1. Let $\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ldots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$

$\Sigma_3$ contains all size 3 columns of 0s and 1s. A string of symbols in $\Sigma_3$ gives three rows of 0s and 1s. Consider each row to be a binary number and let $B = \{ w \in \Sigma_3^* \mid \text{the bottom row of } w \text{ is the sum of the top two rows} \}.$

For example, $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \in B,$ but $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \not\in B.$ Show that $B$ is regular.

2. Let $\Sigma_3$ be the same as in Problem 1. Let $M = \{ w \in \Sigma_3^* \mid \text{the bottom row of } w \text{ is the product of the top two rows} \}.$

For example, $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \in M,$ but $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \not\in M.$ Show that $M$ is not regular.

3. Let $A$ be any language. Define $\text{DROP-ONE}(A)$ to be the language containing all strings that can be obtained by removing one symbol from a string in $A.$

Thus, $\text{DROP-ONE}(A) = \{ xz \mid xyz \in A \text{ where } x, z \in \Sigma^*, y \in \Sigma \}.$

Show that the class of regular languages is closed under the $\text{DROP-ONE}$ operation.

Give both a proof by picture and a more formal proof by construction as in Theorem 1.47.

4. Let $\Sigma = \{0, 1\}.$ Let $WW_k = \{ ww \mid w \in \Sigma^* \text{ and } w \text{ is of length } k \}.$ Show that for each $k$:

(a) No DFA can recognize $WW_k$ with fewer than $2^k$ states.

(b) Some NFA with $O(k)$ states can recognize $\overline{WW}_k,$ the complement of $WW_k.$

Here $O(k)$ means at most $ck$ for some constant $c$ independent of $k.$

5. Let $\Sigma = \{0, 1\}.$

(a) Let $A = \{0^ku0^k \mid k \geq 1 \text{ and } u \in \Sigma^* \}.$ Show $A$ is regular.

(b) Let $B = \{0^ku1u0^k \mid k \geq 1 \text{ and } u \in \Sigma^* \}.$ Show $B$ is not regular.

6. Let $\Sigma = \{0, 1\}$ and let $C_1 = \{ ttr \mid t \in \Sigma^* \text{ and } r \in 0^*10^*, \text{ where } |t| = |r| \}.$ The notation $|x|$ means the length of string $x.$ Show that $C_1$ is a CFL, by giving a CFG and by giving a PDA.

You do not need to prove that your solutions work, but please give comments to assist the grader. (Hint: This problem is tricky but not complicated. It has a CFG with three rules.)

7\* ($\star$ means optional) For any language $A,$ let $RC(A) = \{ yx \mid xy \in A \}.$

Show that the class of regular languages is closed under the $RC$ operation.