Notes 8.370/18.435 Fall 2022

Lecture 15 Prof. Peter Shor

This year (2022), three experimentalists, Alain Aspect, John Clauser and Anton Zeilinger, were awarded the Nobel Prize for experiments showing that quantum mechanics really was non-local. They did their experiments using quantum optics. I want to describe one of these experiments, the GHZ experiment, performed by Zeilinger. However, first I need to explain some basic facts about quantum optics.

What we will do first is explain interferometers, and then explain the Elitzur-Vaidman bomb test. This is a very non-intuitive thought experiment (which has actually been carried out, although not with bombs). It was discovered in 1993, but it only uses quantum mechanics that was known 60 years earlier.

The premise of the test is that you have a lot of bombs. Their fuses are so sensitive that if a single photon strikes them, the bomb goes off. However, some bombs are missing fuses. . .if you shine a photon on the place where the fuse should be, the photon simply passes by unchanged. Your task is to identify some bombs which have intact fuses without exploding them (it's okay if you end up exploding some others).

Classically, it seems absolutely impossible. If you look to see whether there is a fuse there, the bomb explodes. However, if you don't look, you can't tell whether the bomb has a fuse or not. Quantum mechanically, however, it can be done. This is the *Elitzur-Vaidman bomb test experiment*, and it is one of a class of experiments called *interaction-free measurement.*

The key ingredient in this test is an interferometer. An interferometer is an experiment with (in the simplest case) two beam splitters, two mirrors, and two photodetectors. (See Figure.) The beam splitter is like a unitary transform. Assuming there's only

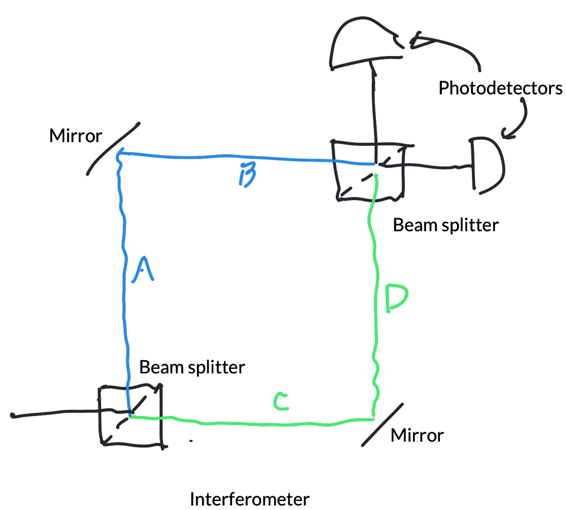

Figure 1: Elitzur Vaidman bomb detector experiment

one photon, this photon can be in a superposition of the horizontal and vertical inputs, and gets transformed into a superposition of the horizontal and vertical outputs. We will take the horizontal input and output to be the state $|\,0\rangle$ and the vertical input and output to be the state $|\,1\rangle$. The matrix that gives the action of the beam splitter is

$$M = \frac{1}{\sqrt{2}} \left(\begin{array}{cc} 1 & i \\ i & 1 \end{array} \right),$$

This isn't the only way that a beam splitter can be described. By changing the bases, you can make it look much more like a Hadamard gate (do this as an exercise). However, this representation has the advantage of treating the vertical and horizontal paths on an equal footing.

Thus, suppose we put a photon in the horizontal input. After it goes through the first beam splitter, the state of the system is

$$M \left(\begin{array}{c} 1 \\ 0 \end{array} \right) = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 1 \\ i \end{array} \right).$$

The photon now travels along the superposition of paths AB and CD and enters the top right beam splitter. If the two paths are both an integer multiple of the wavelength of the light, the state of the photon going into the second beam splitter is the same as the state of the photon that came out of the first beam splitter, except the horizontal and vertical labels have been switched. Thus, the photon going into the second beam splitter is in the state $\frac{1}{\sqrt{2}} \left(\begin{array}{c} i \\ 1 \end{array} \right)$. Multiplying by M again gives the vector $\left(\begin{array}{c} i \\ 0 \end{array} \right)$, so it comes out horizontally and triggers the rightmost photodetector in the drawing.

You can similarly check that if the photon entered the first beam splitter on the vertical path, it would come out the second beam splitter vertically.

How can we use interferometers? Suppose one of the paths applies a phase $e^{i\theta}$ to the photon that the other does not. One way this can happen is if one of the paths is longer than the other. If the difference between the paths is d, then the longer one will acquire an extra phase of $e^{-id/\lambda}$, where λ is the wavelength of the photon. Michelson and Morley used an interferometer to try to measure the speed of the Earth, because under Newtonian mechanics, the path length would be different for photons moving perpendicular and parallel to the Earth's motion. However, because of Einstein's theory of relativity, the path lengths were exactly the same, and the experiment yielded a value of 0 for the Earth's motion. More recently, LIGO is using an interferometer to detect gravity waves—here the path length varies because gravity waves stretch and shrink space-time.

So what is the Elitzur-Vaidman bomb detection experiment? Suppose you have a factory that makes bombs. Each of these bombs has a fuse that is so sensitive that the bomb explodes if a single photon hits it. However, some of these bombs are defective—they are missing fuses. Your mission is to find some bombs that are guaranteed not to be defective. With classical physics, this is clearly impossible; if you shine a photon on a fuse to see whether it's there, the bomb explodes (unless you "cheat" by doing something like weighing the bombs). However, quantum mechanically, you can solve this problem.

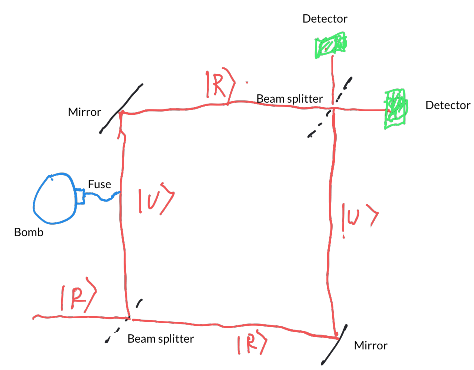

Figure 2: GHZ experiment. The R implements the quantum gate $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$.

What you can do is position a bomb so that the fuse overlaps the 'A' arm of the interferometer. Suppose the bomb is defective. Then, the photon behaves as it would when both the AB and CD paths are unobstructed, so it always triggers right photodetector.

Now, suppose you have a bomb. By exploding, the bomb measures the path the photon took. The photon is in a superposition of taking the AB path and the CD path. So if the bomb doesn't explode, then the state collapses and the photon is on the CD path. When it hits the second beam splitter, it comes out in an equal superposition of the vertical and horizontal exists, so with probability $\frac{1}{2}$, it triggers each of the photodetectors.

Thus, if the fuse was defective, the rightmost photodetector is always triggered, while if the fuse was good, the bomb explodes $\frac{1}{2}$ of the time, the top photodetector is triggered $\frac{1}{4}$ of the time, and the rightmost photodetector is triggered $\frac{1}{4}$ of the time.

So when the top photodetector is triggered, you know that the fuse is not defective.

This experiment has actually been carried out, although not with real bombs.

There is a variation on this experiment where you can make the bomb will explode an arbitrarily small fraction of the time (not 0) and still get a good yield of bombs guaranteed to be non-defective. It's quite a bit more complicated, so we won't go into it.

There are some more quantum optical elements we need to discuss before we can explain the GHZ experiment. One of them is a polarizing beam splitter. This is a device that transmits horizontally polarized photons and reflects vertically polarized photons.

Another one of them is a parametric downconverter. This is a crystal which, when you send a beam of light with frequency ν, you get as output, in addition to a slightly attenuated beam of light with frequency ν, two beams of light with frequency $\nu/2$. What is happening (to give a slightly simplified explanation) is that some of the photons of frequency ν are splitting into pairs of photons of frequency $\nu/2$. Recall that a photon's energy is proportional to its frequency, so energy is conserved. Further, these two photons are entangled, being in the EPR state $\frac{1}{\sqrt{2}}(|\leftrightarrow\updownarrow\rangle - |\updownarrow\leftrightarrow\rangle)$.

With just a parametric downconverter, beam splitters, and polarizing beam splitters, it is fairly straightforward to set up an experiment that shows violations of Bell's inequality. I'll let you figure out how this works.

The experiment becomes much more difficult to implement if you want to close all the loopholes. You have to prove beyond all doubt that the detectors aren't communicating somehow; the way to do that is to make sure the settings of the detectors are changed fast enough that the information from one about the setting of one can't reach the other in time, even if it's transmitted at the speed of light. This requires quite a bit more sophistication on the experimenter's part.

However, for the GHZ experiment, we need three entangled photons, and parametric downconverters only produce two. How can we possibly entangle three photons with just a parametric downconverter and the optical elements we know?

What Pan, Bouwmeester, Daniell, Weinfurter and Anton Zeilinger did was to use the apparatus in the figure below. You need to illuminate the parametric downconverter at a high enough intensity so that you will occasionally get two simultaneous downconversions (creating four photons in two entangled pairs), but not a high enough intensity that there are many simultaneous downconversions of three photons.

Now, you only count instances where four detectors (at the end of each of the four paths) register photons. There are two entangled photons, and each of these entangled pairs sends one photon down path a and the other down path b.

For the Detector 1, you can only detect horizontal photons, because only those pass through the polarizing filter. This means that the other photon that went down path a was a vertical one, as otherwise it would have also gone through the polarizing beam splitter and into Detector 1. This photon goes through the gate R and turns into $\frac{1}{\sqrt{2}}(|\leftrightarrow\rangle + |\updownarrow\rangle)$. Now, before the photon goes through the gate R, there must be an equal number of horizontally and vertically polarized photons (because they form two EPR pairs). This means the photons must be in the state $|\leftrightarrow\updownarrow\leftrightarrow\updownarrow\rangle$ or $|\leftrightarrow\updownarrow\updownarrow\leftrightarrow\rangle$ And in fact, it is easy to show that both of these states have the same phase.

Now, there must be two photons that approach the central polarizing beam splitter, and these two photons must have the same polarization (otherwise they would both end up at only one of the two central detectors). Thus, the first three photons must either be in the state $|\leftrightarrow\leftrightarrow\leftrightarrow\rangle$ or $|\leftrightarrow\updownarrow\updownarrow\rangle$. Since we know that the second photon was originally $|\updownarrow\rangle$ before it went through the gate R, and there were two of each horizontal and vertical polarizations, the only two possibilities are. $|\leftrightarrow\leftrightarrow\leftrightarrow\updownarrow\rangle$ or $|\leftrightarrow\updownarrow\updownarrow\leftrightarrow\rangle$. Thus, the state of the last three photons are

$$\frac{1}{\sqrt{2}}(|\leftrightarrow\leftrightarrow\updownarrow\rangle + |\updownarrow\updownarrow\leftrightarrow\rangle),$$

which is essentially a GHZ state. If we apply a NOT gate to the last of these photons,

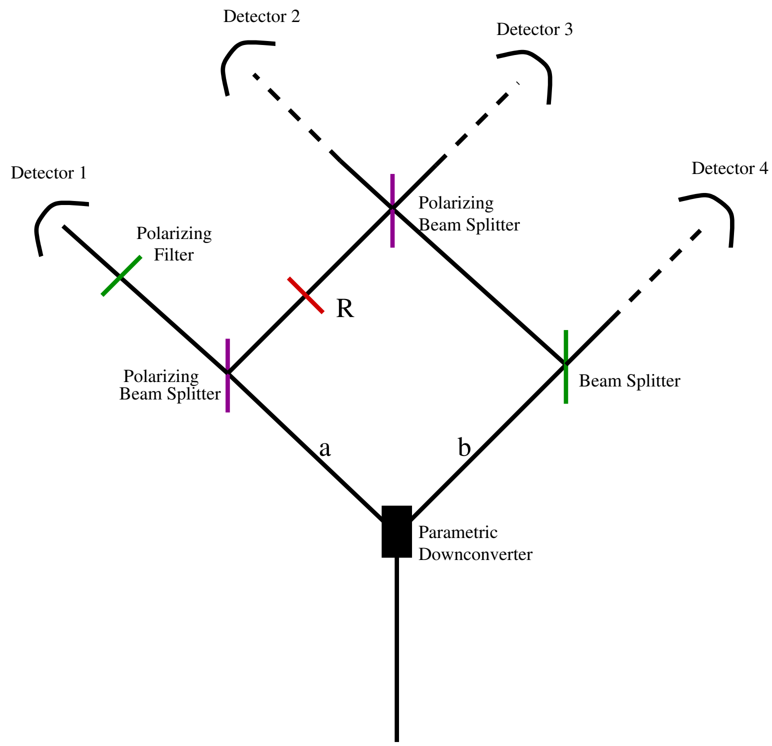

Figure 3: Mach-Zehnder Interferometer

we get

$$\frac{1}{\sqrt{2}}(|\leftrightarrow\leftrightarrow\leftrightarrow\rangle + |\updownarrow\updownarrow\updownarrow\rangle),$$

We still need to add some quantum optics elements before the detectors, in the space where the dashed lines are, to measure the GHZ states in the correct basis. Note that this means we don't actually have to apply a NOT gate to the photon which will hit Detector D, since we can adjust these elements to compensate for the lack of a NOT gate.