18.435/2.111 Homework # 9

Due Thursday, November 30

1: Define

$$\sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad \sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

Show that

$$e^{i\frac{\omega t}{2}\sigma_z}\,\sigma_\pm\,e^{-i\frac{\omega t}{2}\sigma_z} = e^{\pm i\omega t}\,\sigma_\pm$$

Read Section 8.3.5

2: Do Exercise 8.20 in Nielsen and Chuang.
3: Do Exercise 8.23 in Nielsen and Chuang.

4: Suppose that there is some quantum operation Φ that preserves two distinct non-orthogonal states. That is, if $|\,v\rangle$ and $|\,w\rangle$ are unit vectors satisfying

$$\begin{aligned} \langle v|w\rangle &\neq 0 \\ |\langle v|w\rangle| &\neq 1 \end{aligned}$$

and

$$\begin{aligned} \Phi(|\,v\rangle\langle v\,|) &= |\,v\rangle\langle v\,| \\ \Phi(|\,w\rangle\langle w\,|) &= |\,w\rangle\langle w\,|, \end{aligned}$$

prove that Φ is the identity map.

5a. Suppose that somebody flips a coin and gives you a qubit which is in a state given by either density matrix ρ_1 or ρ_2, each with probability $\frac{1}{2}$. Find the projective measurement which predicting which state it is in with the lowest overall probability of error.
Hint: Write down the measurement explicitly and calculate the error probability. Remember that if $\rho_1 = |\,v\rangle\langle v\,|$ and $\rho_2 = I - |\,v\rangle\langle v\,|$, the error should be 0, and if $\rho_1 = \rho_2$, the error should be $\frac{1}{2}$.

5b. Will using a POVM give a lower error probability in problem 5a?