

## 18.435/2.111 Homework # 7

Due Thursday, November 10

**1:** This is problem 6.1 in Nielsen and Chuang. Suppose that  $x_1, \dots, x_N$  is a database of numbers held in memory, as in Section 6.5. Show that only  $O(\log(N)\sqrt{N})$  accesses to the memory are required on a quantum computer, in order to find the smallest element on the list, with probability at least one-half.

**2:** Suppose that you have a qubit and want to perform the POVM with elements

$$\begin{pmatrix} 1/2 & 1/4 \\ 1/4 & 1/8 \end{pmatrix} \quad \begin{pmatrix} 1/2 & -1/4 \\ -1/4 & 1/8 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 \\ 0 & 3/4 \end{pmatrix}$$

You can achieve the same probabilities of outcomes, for any given input state  $|\psi\rangle$ , by adding a qubit in state  $|0\rangle$ , applying a two-qubit unitary transformation  $U$ , and then applying the projective measurement in the computational basis. (And if you wish, you can duplicate this POVM exactly by subsequently preparing a qubit with the correct outcome.) Give a unitary transformation  $U$  that will achieve this result.