

18.435/2.111 Homework # 7

Due Thursday, November 10

1: This is problem 6.1 in Nielsen and Chuang. Suppose that x_1, \dots, x_N is a database of numbers held in memory, as in Section 6.5. Show that only $O(\log(N)\sqrt{N})$ accesses to the memory are required on a quantum computer, in order to find the smallest element on the list, with probability at least one-half.

2: Suppose that you have a qubit and want to perform the POVM with elements

$$\begin{pmatrix} 1/2 & 1/4 \\ 1/4 & 1/8 \end{pmatrix} \quad \begin{pmatrix} 1/2 & -1/4 \\ -1/4 & 1/8 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 \\ 0 & 3/4 \end{pmatrix}$$

You can achieve the same probabilities of outcomes, for any given input state $|\psi\rangle$, by adding a qubit in state $|0\rangle$, applying a two-qubit unitary transformation U , and then applying the projective measurement in the computational basis. (And if you wish, you can duplicate this POVM exactly by subsequently preparing a qubit with the correct outcome.) Give a unitary transformation U that will achieve this result.