18.435/2.111 Homework # 1

Due Thursday, September 21

1: Show how you would make a circuit out of Toffoli gates which, when the input is guaranteed to have exactly one 1, finds the position of the 1. That is, for inputs 1000, 0100, 0010, or 0001, the output should be 00, 01, 10, 11, respectively, and the input should be erased. Any extra bits you add should take fixed values for these four inputs in both the input and the output. If the input has more (or less) than one 1, I don't care what the circuit does, and the extra bits don't have to take constant values. Either give the circuit, or describe it in enough detail so that the TA would easily be able to write it down using your description. How many Toffoli gates does your circuit have?

2: Show that you cannot build such a circuit out of one- and two-bit reversible gates.

3: Suppose that you relax the requirements so you are allowed to keep the input. That is, 1000 must go to 100000, 0100 must go to 010001, etc. Can you then build it out of CNOT gates with the extra bits being fixed values for these four inputs?

4: Exponentiate the appropriate matrix to find the unitary matrix which performs a $120°$ rotation around the axis $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ for a spin-$\frac{1}{2}$ particle. Verify that this matrix is correct by showing what it does to spin $\frac{1}{2}$ states in the x-, y-, and z- directions?

5: For a spin $\frac{1}{2}$ particle, show that no unitary matrix will fix spin up and down in both the x and z directions, and interchange spin down and spin up in the y direction.