Testing high-d distributions with subcube conditioning

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S. Ristic (Penn) **D. Chakrabarty (Dartmouth)** **C. Seshadhri (UC Santa Cruz)**

Brief Outline:

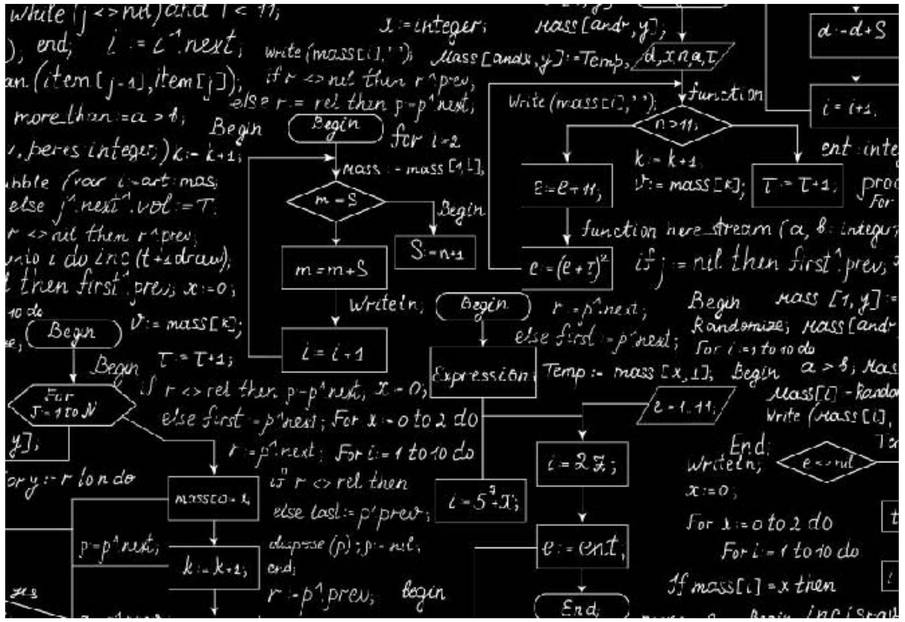
1. Motivation: distribution testing and the curse of dimensionality.
2. Property testing of distributions.
3. New results.

Main Premise:
Testing high-d discrete distributions
by *algorithmically* interacting with the distribution.

Types of D*iscrete, High-d* distributions

Coming from Nature

Software or Generative Model



Discrete Distribution from Enormous collection

Types of D*iscrete, High-d* distributions

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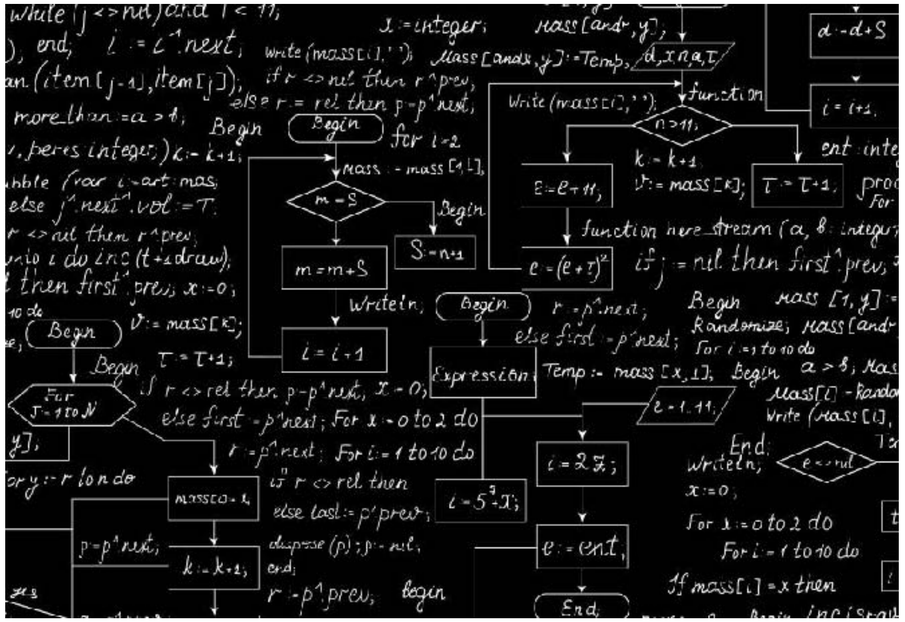

Discrete Distribution from Enormous collection

Algorithmic question: How to extract information from these high-d distributions?

Which is uniform on $\{0,1\}^{15}$?

$\mathcal{D}_1$

0	1	0	1	0	1	1	0	0	1	0	1	1	0	0
1	0	1	1	0	1	1	1	1	0	0	1	1	1	1
0	0	0	0	1	1	0	1	1	0	0	1	0	1	0
1	1	1	0	0	1	1	1	0	0	0	0	0	1	1
1	1	0	1	1	1	0	1	1	1	1	0	1	1	1
0	0	0	0	1	1	0	1	1	0	0	1	0	0	0
1	0	1	1	0	1	1	1	1	1	1	1	0	0	1
1	1	1	0	1	0	0	1	0	1	0	0	0	1	0

$\mathcal{D}_2$

0	1	1	1	0	1	0	1	0	0	0	0	1	0	0
1	1	1	0	0	1	0	1	0	1	0	1	0	0	1
1	1	0	1	0	1	0	0	1	0	0	0	1	0	0
1	1	0	1	0	1	1	0	0	1	0	1	1	0	0
0	1	1	0	1	1	0	0	1	1	1	0	0	1	0
1	1	0	1	1	1	0	0	0	1	0	0	1	1	0
0	1	0	0	0	0	1	1	0	1	0	1	1	1	1
1	0	1	0	0	1	0	0	1	1	1	0	0	0	0
0	1	0	0	1	0	0	1	0	1	0	0	1	0	1

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0	1	0	1	0	1	1	0	0	1	0	1	1	0	0
1	0	1	1	0	1	1	1	1	0	0	1	1	1	1
0	0	0	0	1	1	0	1	1	0	0	1	0	1	0
1	1	1	0	0	1	1	1	0	0	0	0	0	1	1
1	1	0	1	1	1	0	1	1	1	1	0	1	1	1
0	0	0	0	1	1	0	1	1	0	0	1	0	0	0
1	0	1	1	0	1	1	1	1	1	1	1	0	0	1
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1	1	1	0	0	1	0	1	0	1	0	1	0	0	1
1	1	0	1	0	1	0	0	1	0	0	0	1	0	0
1	1	0	1	0	1	1	0	0	1	0	1	1	0	0
0	1	1	0	1	1	0	0	1	1	1	0	0	1	0
1	1	0	1	1	1	0	0	0	1	0	0	1	1	0
0	1	0	0	0	0	1	1	0	1	0	1	1	1	1
1	0	1	0	0	1	0	0	1	1	1	0	0	0	0
0	1	0	0	1	0	0	1	0	1	0	0	1	0	1

$$\boldsymbol{x} \sim \mathcal{D}_2 : \sum_{i=1}^{15} \boldsymbol{x}_i \quad \textbf{even}$$

How to efficiently extract information from a distribution?

Question 1: Is it uniform/the distribution you are expecting? [GR'00, Paninski'08]

Question 2: Are two distributions the same? [BFRSW '00]

Question 3: Estimate properties of distributions … [VV'11]

How to efficiently extract information from a distribution?

Question 1: Is it uniform/the distribution you are expecting? [GR'00, Paninski'08]

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Question 3: Estimate properties of distributions … [VV'11]

Algorithmic Message: usable information in collision statistics,
often much more efficient than learning.

Birthday Paradox

Theorem (Paninski'08): Testing whether a distribution $\mathcal{D}$ is the uniform dist. (or any fixed reference) over Ω can be done with $\sqrt{|\Omega|}$ samples, and is tight.

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Theorem (Paninski'08): Testing whether a distribution $\mathcal{D}$ is the uniform dist. (or any fixed reference) over $\{\pm 1\}^d$ can be done with $2^{d/2}$ samples, and is tight.

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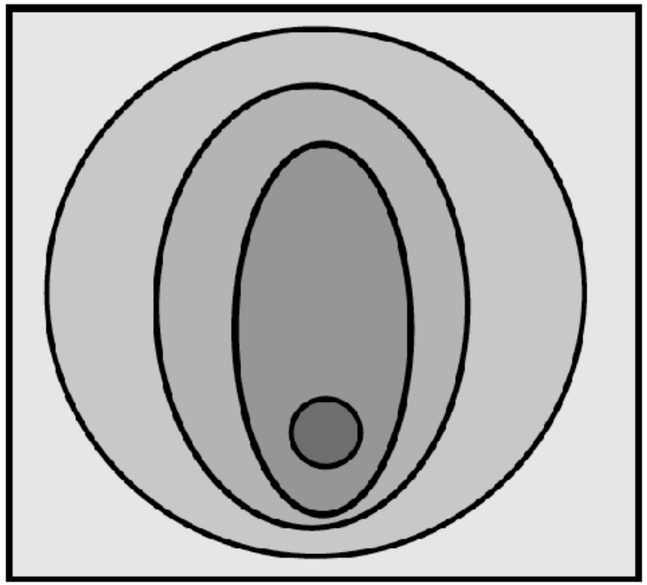
Theorem (Paninski'08): Testing whether a distribution $\mathcal{D}$ is the uniform dist. (or any fixed reference) over $\{\pm 1\}^d$ can be done with $2^{d/2}$ samples, and is tight.

Two Things:

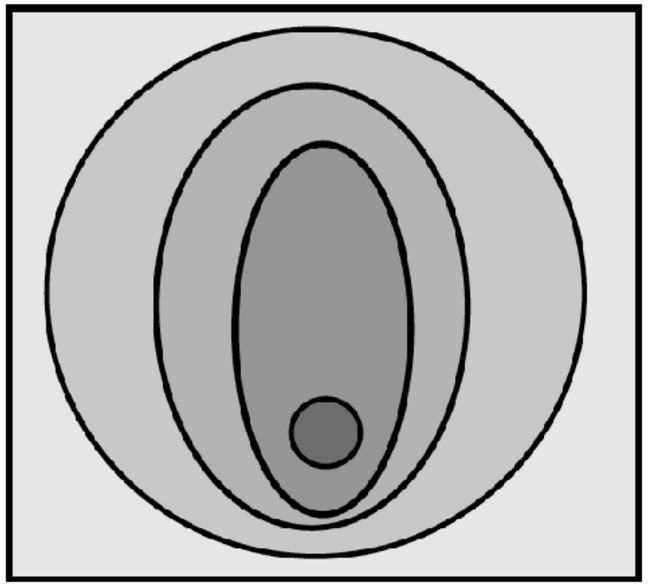
1. Impressive sublinear bound and made it unimpressive.
2. Assigned meaning to coordinates.

Beyond “curse of dimensionality”…



Restricted Inputs
(Ising models, Bayes nets,
small mixtures, monotone dist)

Beyond “curse of dimensionality”…



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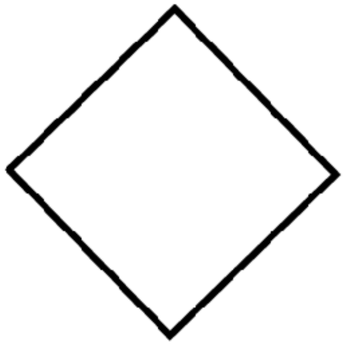
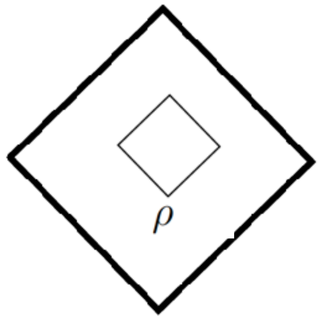


$\{-1,1\}^n$

Stronger Access

This talk's access: Subcube Conditional Sampling [Bhattacharrya, Chakraborty '18]

$\{-1,1\}^n$



$\rho \in \{-1,*,1\}^n$

x from $\mathcal{D}$
s.t. $\rho_i = x_i$ for $\rho_i \neq *$

$\rho =$	-1	1	-1	*	*	1	1	*	-1	*	1	1	1	1	-1
$x =$	-1	1	-1	**1**	**-1**	1	1	**1**	-1	**-1**	1	1	1	1	-1

Prefix [Adar, Fisher, Levi '25]

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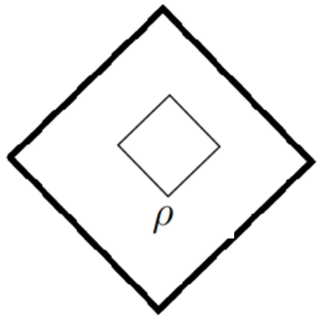

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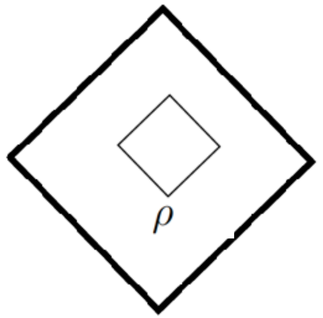
Coordinate [Blanca, Chen, Stefanovic, Vigoda ’23]

$\{-1,1\}^n$



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$x =$	-1	1	-1	1	**-1**	1	1	1	-1	-1	1	1	1	1	-1

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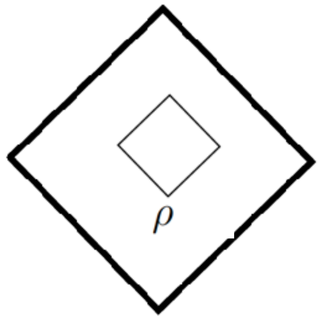
~~**Coordinate**~~ [Blanca, Chen, Stefanovic, Vigoda ’23]

(General) Conditioning [Canonne, Ron, Servedio’12;
Chakraborty, Fisher, Goldhirsh, Matsliah ’12]

$\rho \in \{-1, *, 1\}^n$

$\{-1,1\}^n$



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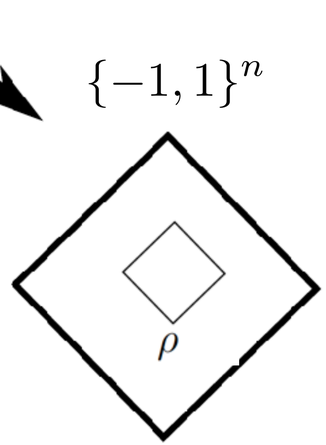
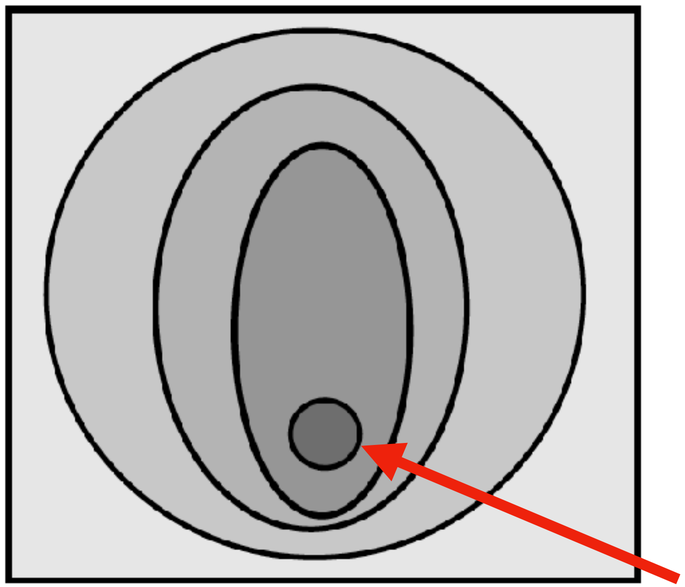


x from $\mathcal{D}$
s.t. $\rho_i = x_i$ for $\rho_i \neq *$

Algorithmic Message: usable information in marginals of distribution, but only after conditioning on random subcubes.

What we can currently do…

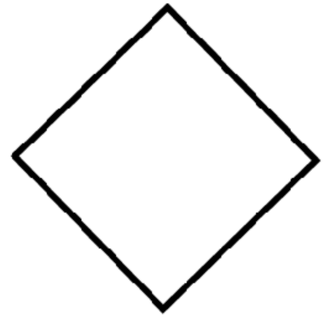
Property	Complexity	$[m]^n$
Uniformity	$\tilde{\Theta}(\sqrt{n}/\epsilon^2)$	$\tilde{O}(\mathrm{poly}(m)\sqrt{n}/\epsilon^2)$ [CM'23]
Junta Testing	$\tilde{\Theta}((k+\sqrt{n})/\epsilon^2)$	
Junta Learning	$\tilde{\Theta}(k/\epsilon^2)\cdot\log n$	
Monotonicity	$\tilde{\Theta}(n/\epsilon^2)$	



Restricted Inputs

Product distributions

subcube conditioning
=
samples



$\{-1,1\}^n$

Subcube Conditioning

Product Distributions are much simpler.

$\mathcal{D}_1$

1	0	0	0	0	0	1	1	0	1	1	0	1	1	1
0	0	1	0	1	0	0	0	1	1	1	1	1	1	0
0	1	1	0	0	1	0	1	0	1	0	0	1	1	1
0	0	1	1	0	0	0	1	1	0	1	0	1	1	0
0	0	0	0	0	0	0	0	1	1	1	1	1	0	1
0	1	0	0	0	0	1	1	1	0	1	0	1	1	1
0	0	0	0	1	0	0	0	0	1	1	1	0	1	1
0	0	0	0	0	0	0	1	0	1	1	1	1	1	1
0	0	0	0	0	0	0	1	1	1	0	1	1	1	0

$\mathcal{D}_2$

0	1	0	1	0	1	1	0	0	1	0	1	1	0	0
1	0	1	1	0	1	1	1	1	0	0	1	1	1	1
0	0	0	0	1	1	0	1	1	0	0	1	0	1	0
1	1	1	0	0	1	1	1	0	0	0	0	0	1	1
1	1	0	1	1	1	0	1	1	1	1	0	1	1	1
0	0	0	0	1	1	0	1	1	0	0	1	0	0	0
1	0	1	1	0	1	1	1	1	1	1	1	0	0	1
1	1	1	0	1	0	0	1	0	1	0	0	0	1	0

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1	0	0	0	0	0	1	1	0	1	1	0	1	1	1
0	0	1	0	1	0	0	0	1	1	1	1	1	1	0
0	1	1	0	0	1	0	1	0	1	0	0	1	1	1
0	0	1	1	0	0	0	1	1	0	1	0	1	1	0
0	0	0	0	0	0	0	0	1	1	1	1	1	0	1
0	1	0	0	0	0	1	1	1	0	1	0	1	1	1
0	0	0	0	1	0	0	0	0	1	1	1	0	1	1
0	0	0	0	0	0	0	1	0	1	1	1	1	1	1
0	0	0	0	0	0	0	1	1	1	0	1	1	1	0

$\Pr[1] = 0.2$ $\Pr[1] = 0.8$

$\mathcal{D}_2$

0	1	0	1	0	1	1	0	0	1	0	1	1	0	0
1	0	1	1	0	1	1	1	1	0	0	1	1	1	1
0	0	0	0	1	1	0	1	1	0	0	1	0	1	0
1	1	1	0	0	1	1	1	0	0	0	0	0	1	1
1	1	0	1	1	1	0	1	1	1	1	0	1	1	1
0	0	0	0	1	1	0	1	1	0	0	1	0	0	0
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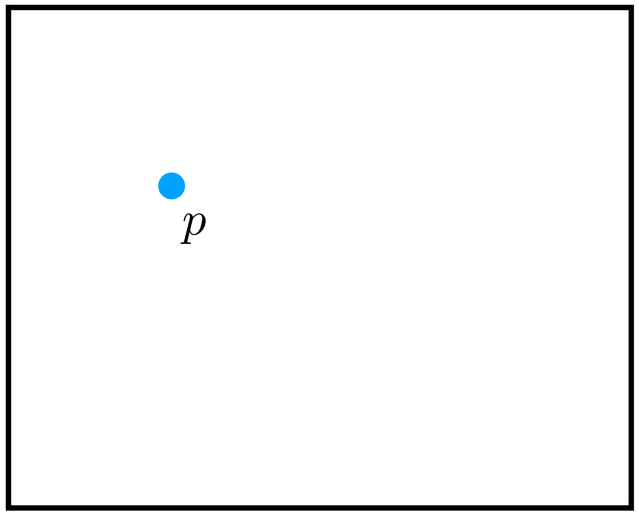
1	1	1	0	1	0	0	1	0	1	0	0	0	1	0

Which is uniform on $\{0,1\}^{15}$?

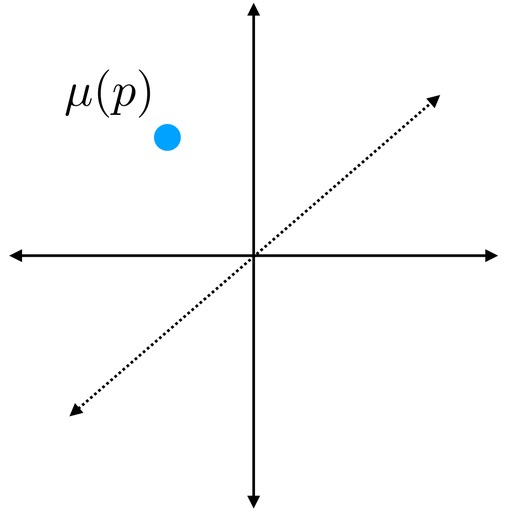
Product Distributions on $\{-1,1\}^n$

$$\mu(p) = (\mu(p)_1, \mu(p)_2, \ldots, \mu(p)_n) = \mathop{\mathbb{E}}_{\boldsymbol{x}\sim p}[\boldsymbol{x}] \in [-1,1]^n$$

bias of each coordinate



Space of distributions in $\{-1,1\}^n$



Space of $\mu(p) \in [-1,1]^n$

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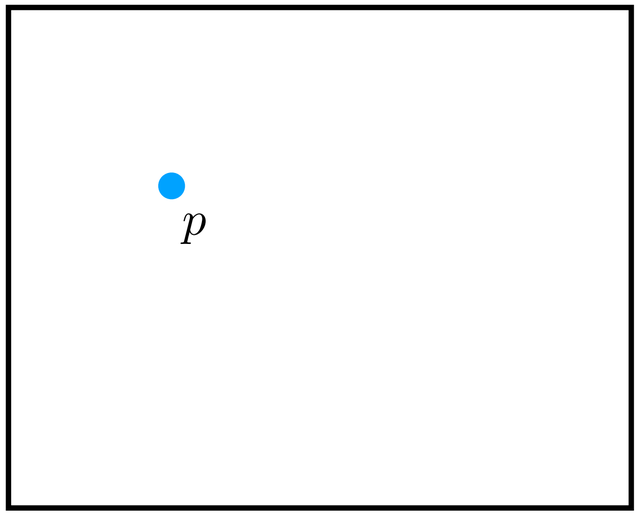
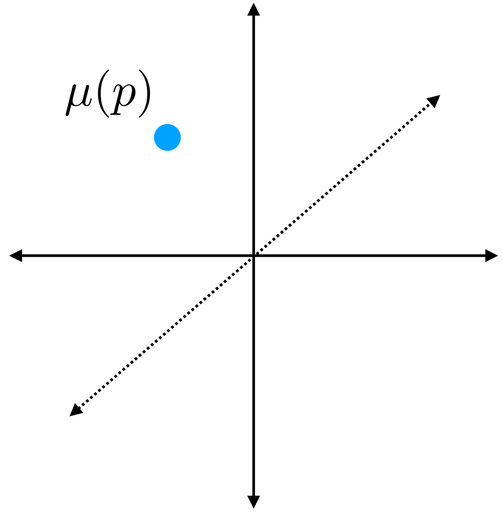


Space of distributions in $\{-1,1\}^n$

Huge reduction in complexity!



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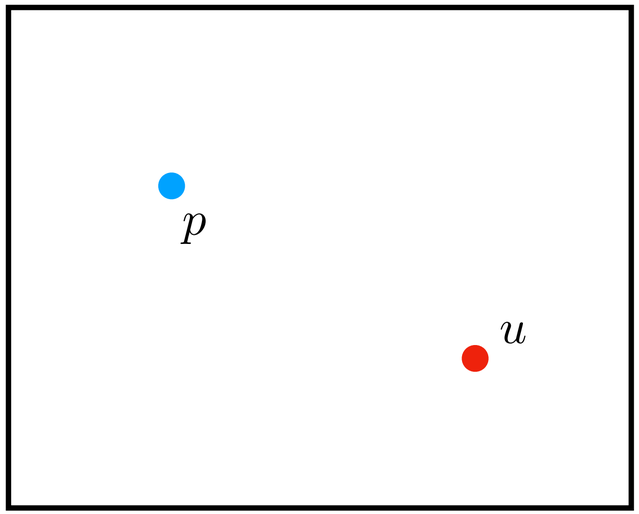

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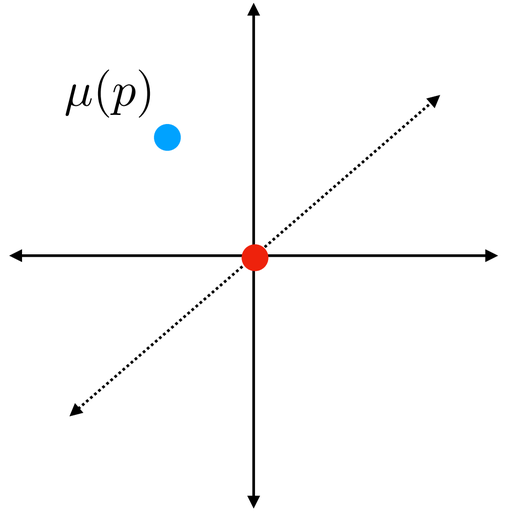

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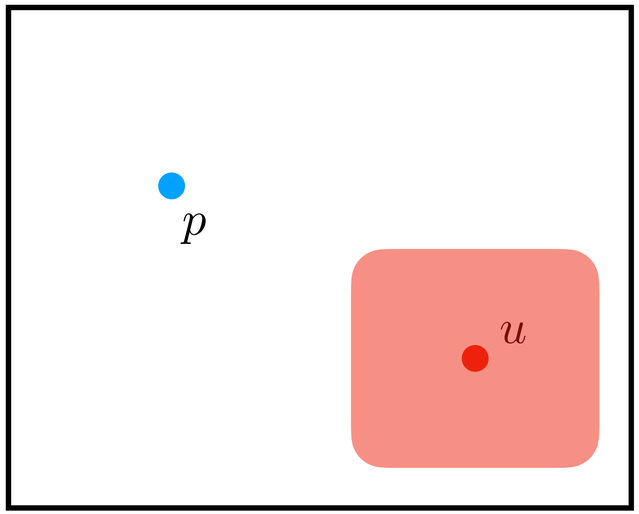

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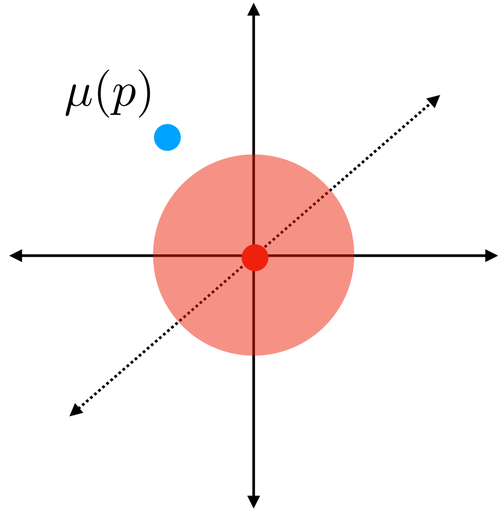

Space of $\mu(p) \in [-1,1]^n$

Simple Claim:

$$\|\mu(p)\|_2 \gtrsim d_{\mathrm{TV}}(p,u)$$

Product Distributions on $\{-1,1\}^n$

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bias of each coordinate

Theorem: There is an algorithm using $O(\sqrt{n}/\epsilon^2)$ samples from a product dist. which can test whether it is uniform or far-from uniform, and is tight.

- Ib implies $\Omega(\sqrt{n}/\epsilon^2)$ subcube queries are necessary.

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General Distributions on $\{-1,1\}^n$

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bias of each coordinate

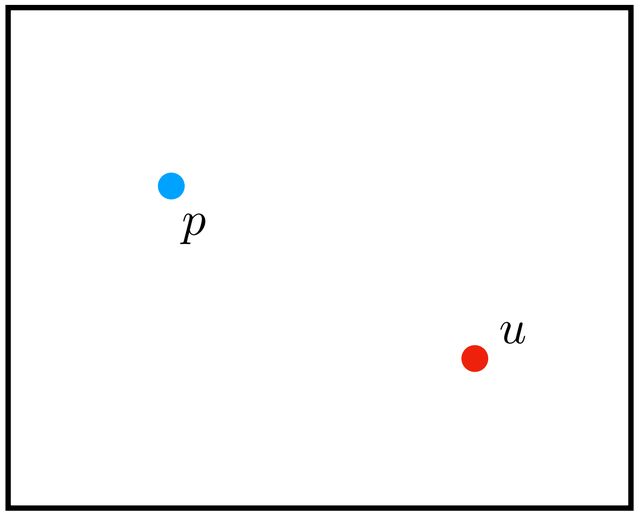
Theorem: There is an algorithm using $\tilde{O}(\sqrt{n}/\epsilon^2)$ ~~samples~~ **subcube queries** from a ~~product~~ **general** dist. which can test whether it is uniform or far-from uniform, and is tight.

- Ib implies $\Omega(\sqrt{n}/\epsilon^2)$ subcube queries are necessary.

~~**Simple Claim:**~~

$$\|\mu(p)\|_2 \gtrsim d_{\mathrm{TV}}(p,u)$$

General Distributions over $\{-1,1\}^n$: mean vectors alone are not good



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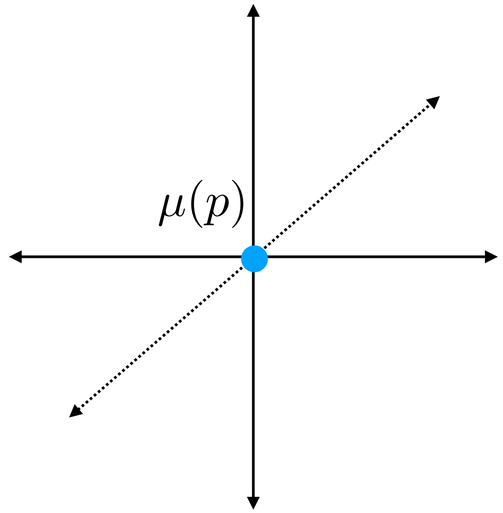

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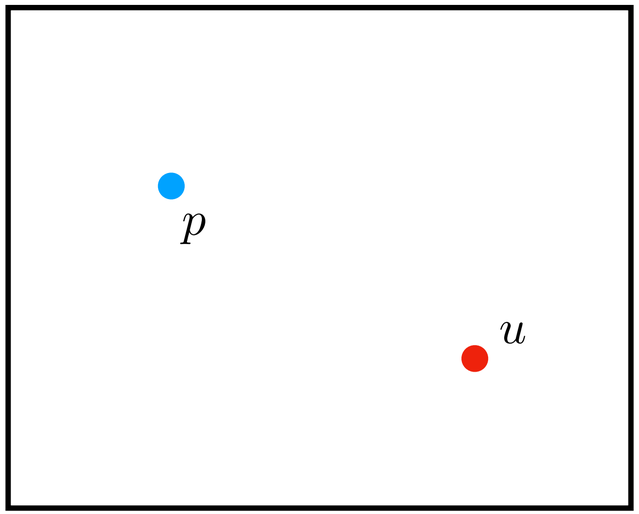

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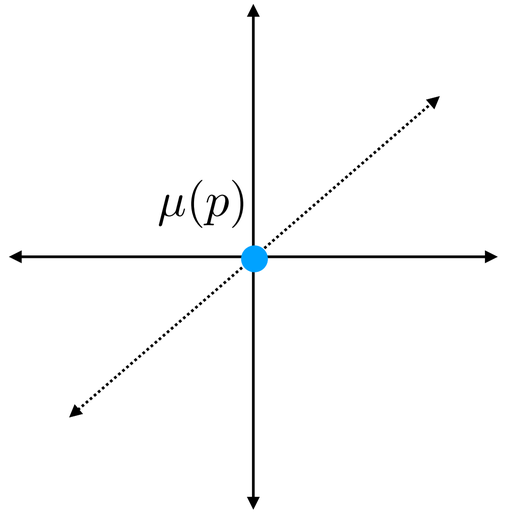

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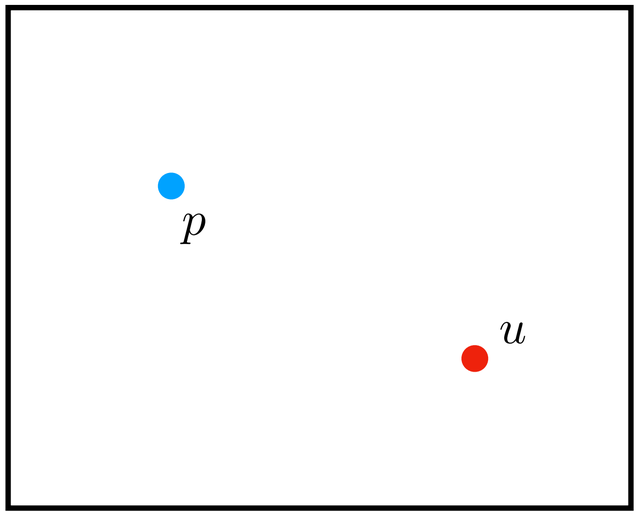
General Distributions over $\{-1,1\}^n$: mean vectors alone are not good



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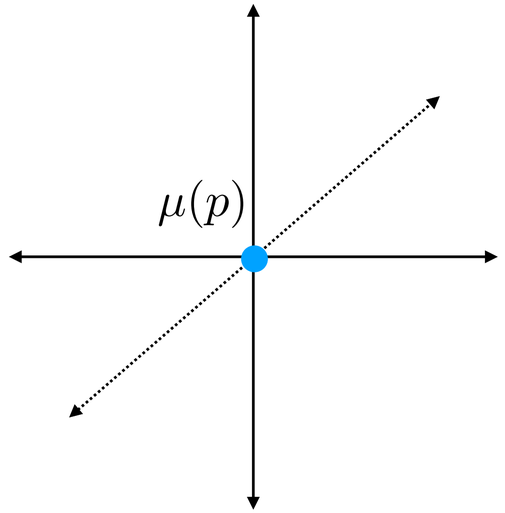

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General Distributions over $\{-1,1\}^n$: mean vectors on random subcubes

Theorem: For any distribution p supported on $\{-1,1\}^n$,

1. Sample 'star-scale' $i \sim [\log_2 n]$, i.e., $\approx 2^i$ stars
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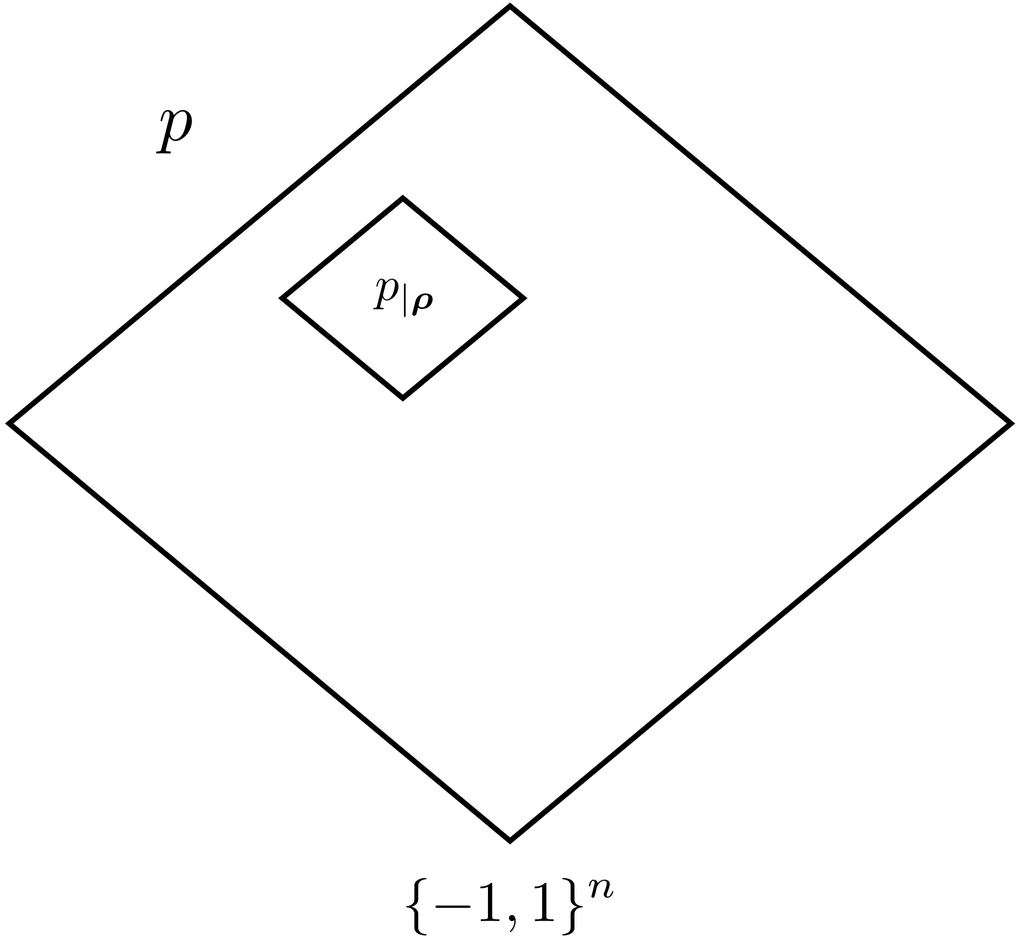
"If all subcubes have unbiased marginals, close to uniform."



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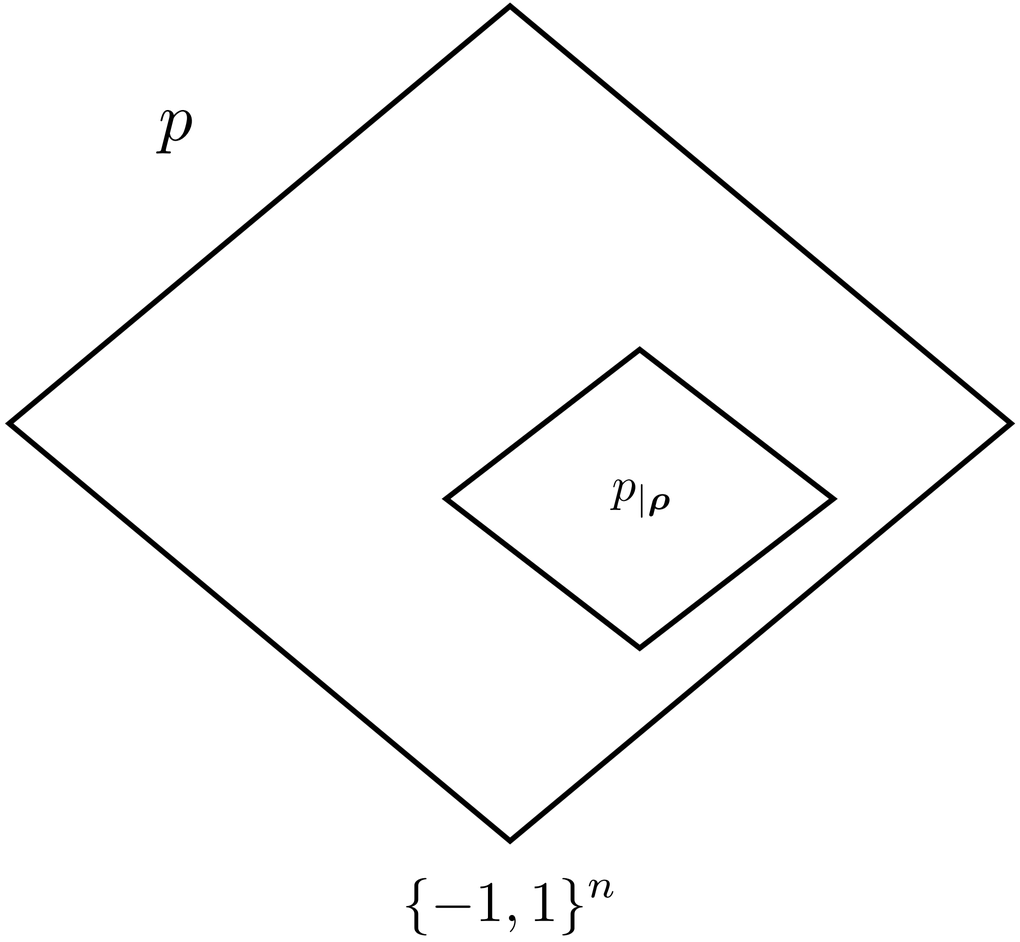

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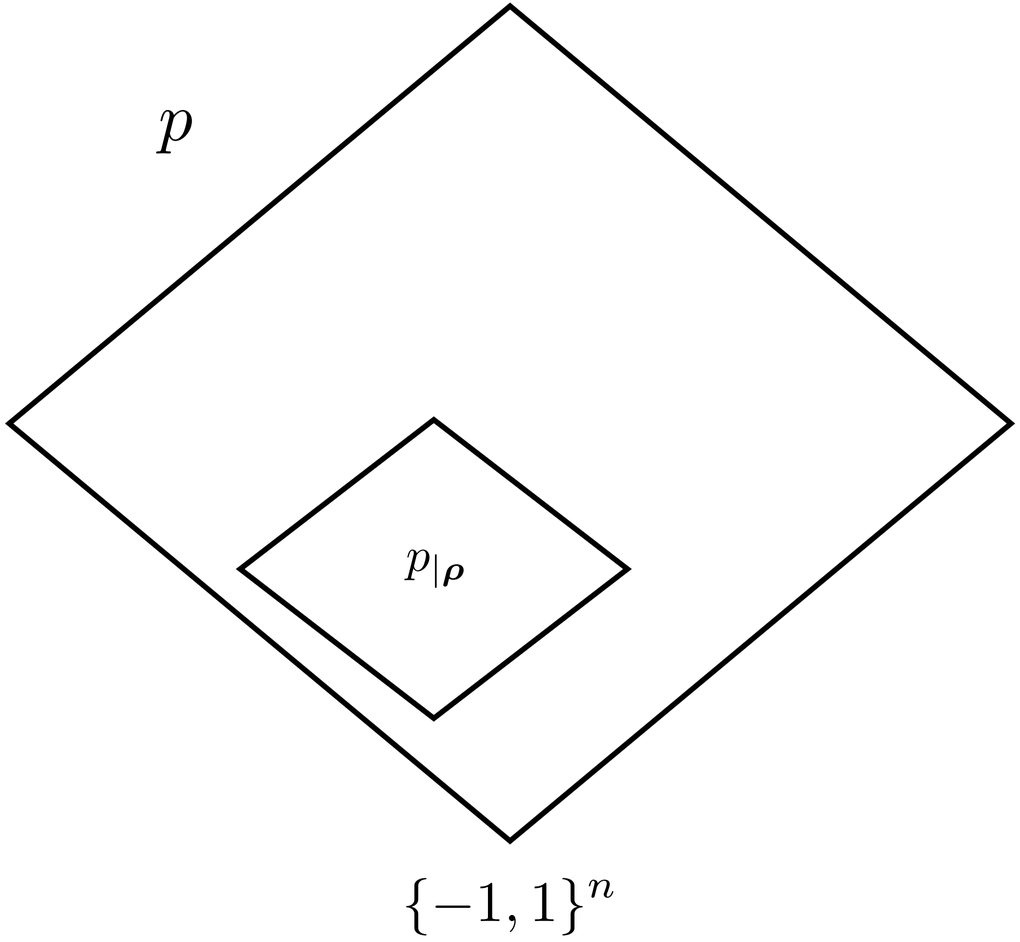

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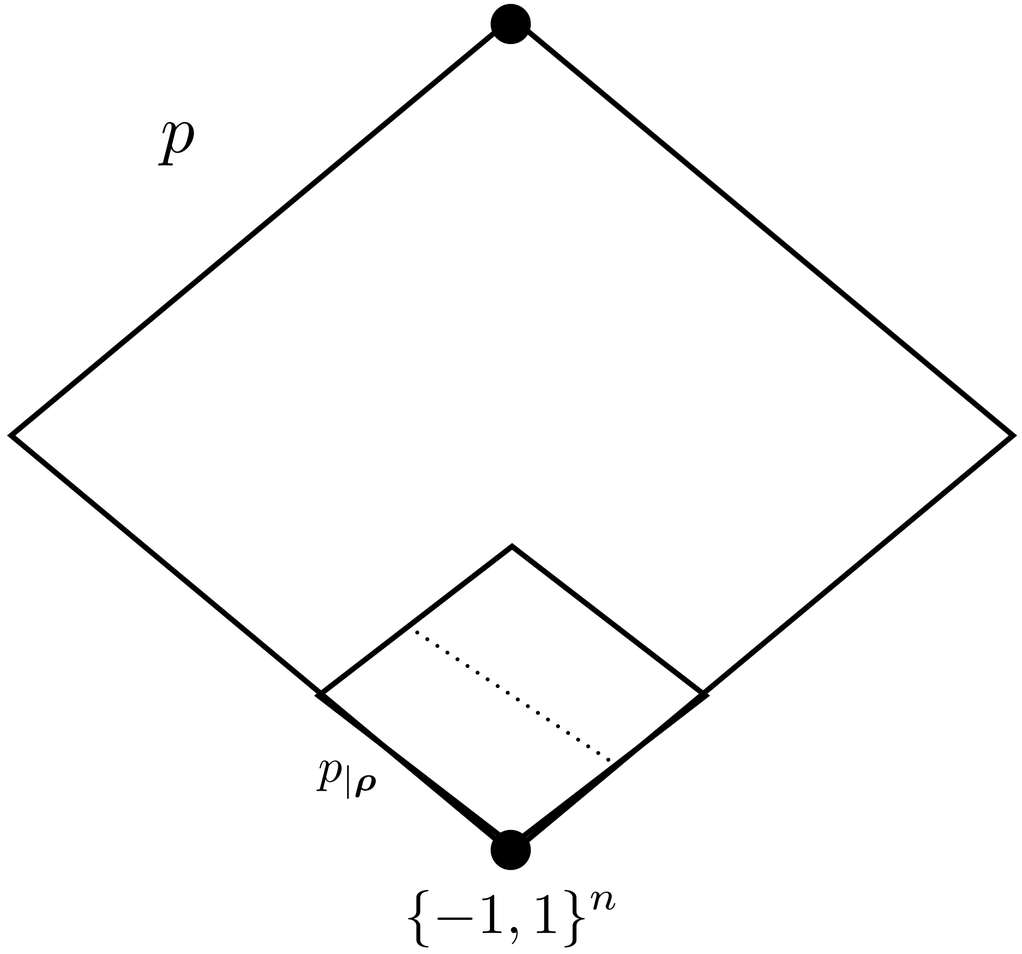
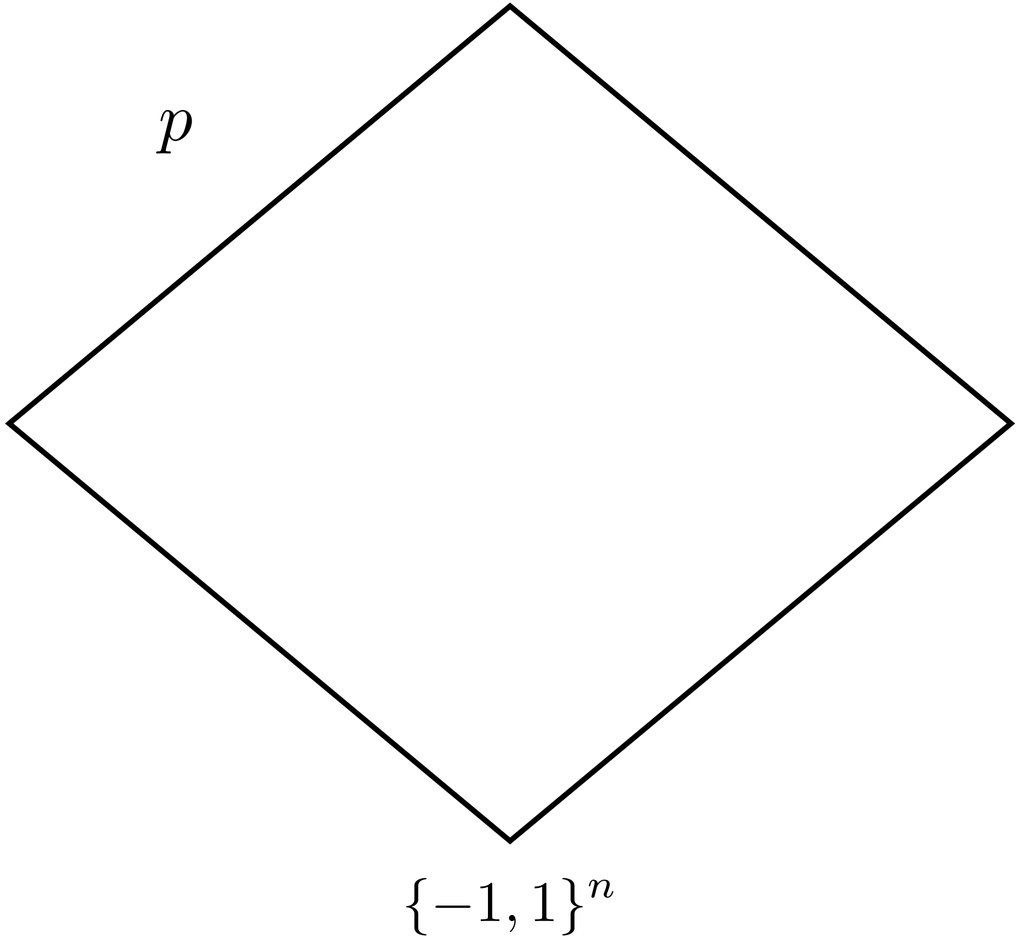


General Distributions over $\{-1,1\}^n$: mean vectors on random subcubes

Examples:

p : most coords uniform, hidden set

$S \subset [n]$ s.t. $\boldsymbol{x} \sim p, \prod_{j \in S} \boldsymbol{x}_j = 1$



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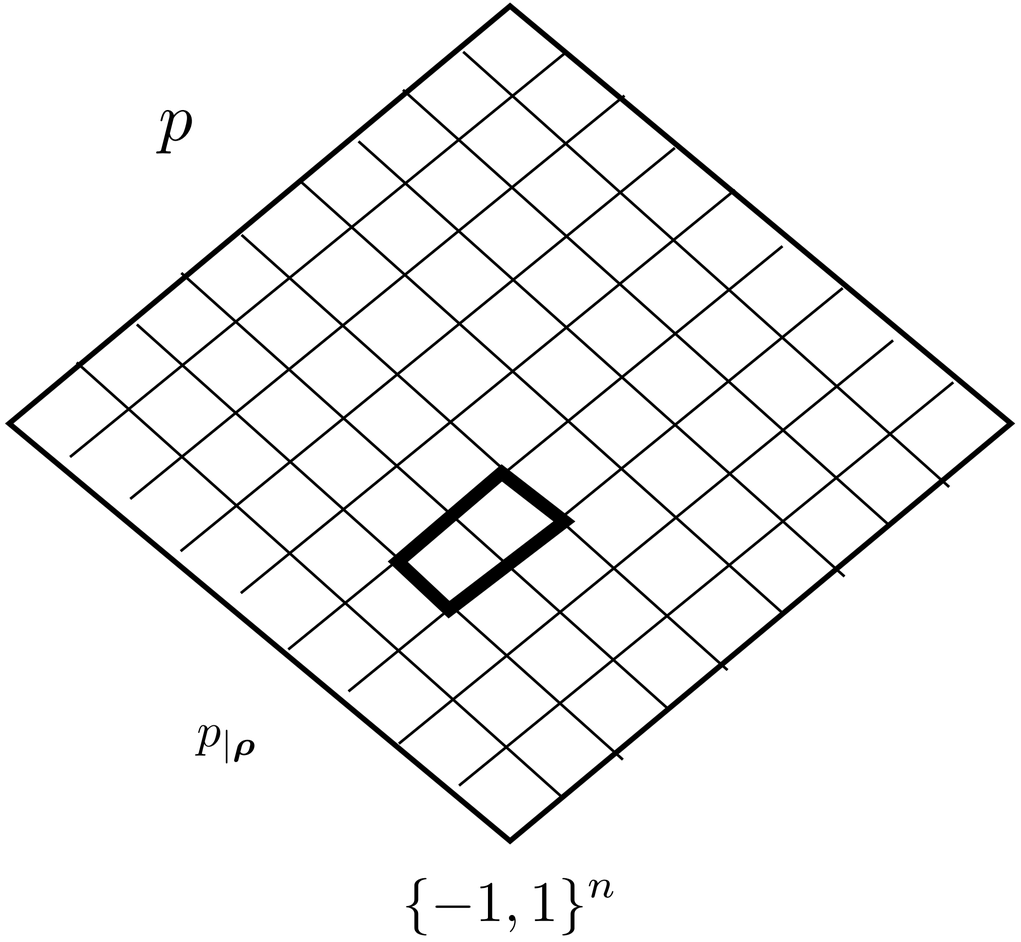

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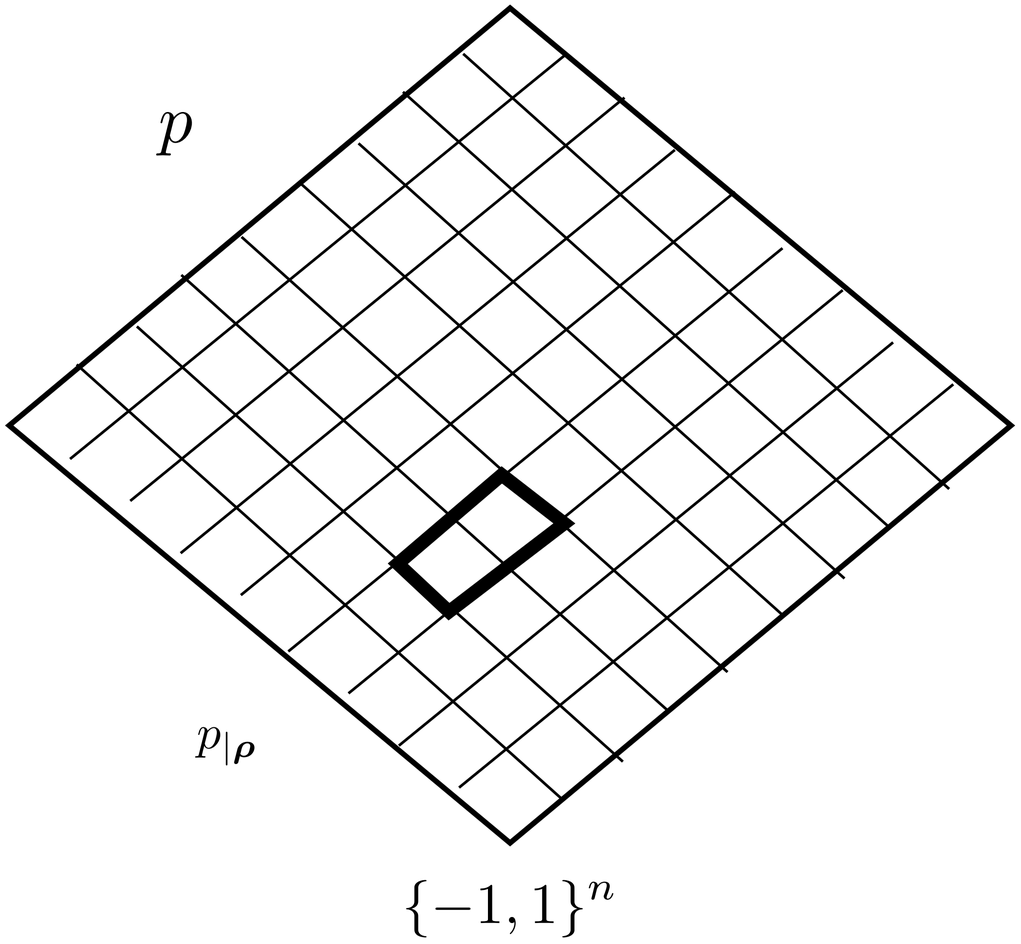
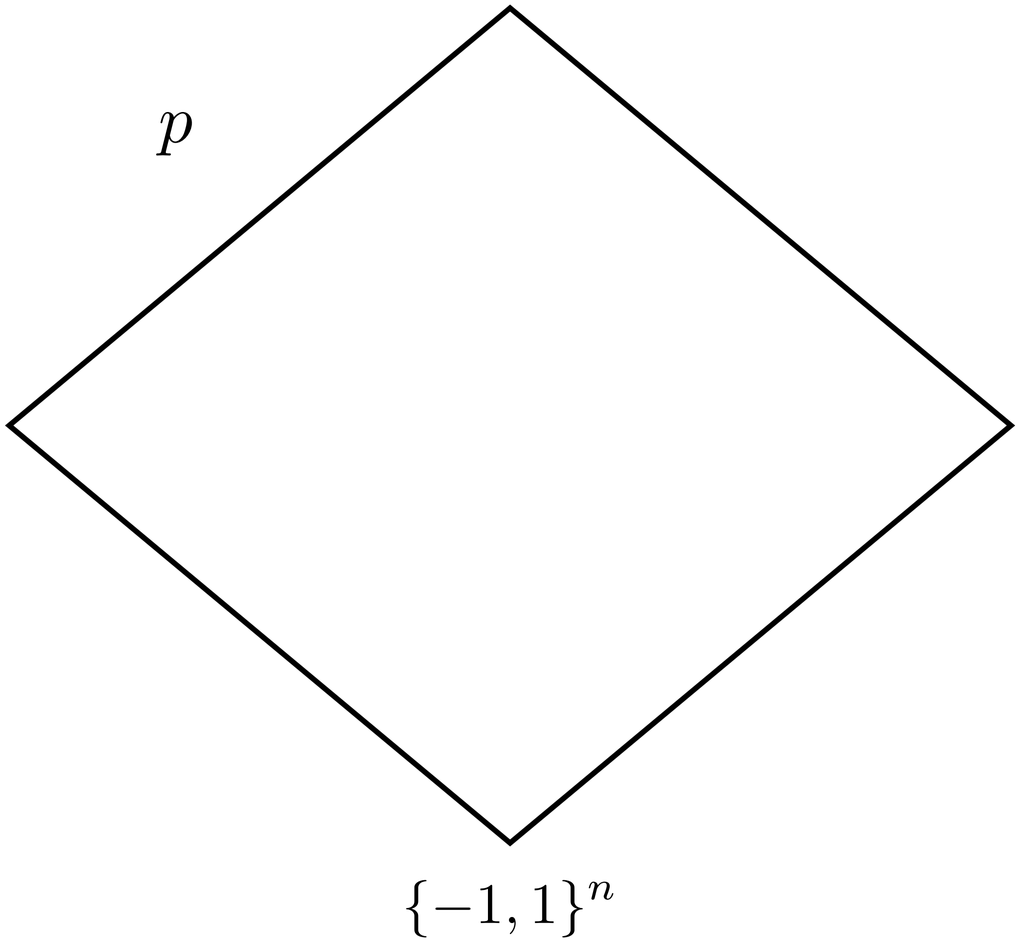


Why should something like this be true?

A distribution is uniform iff:



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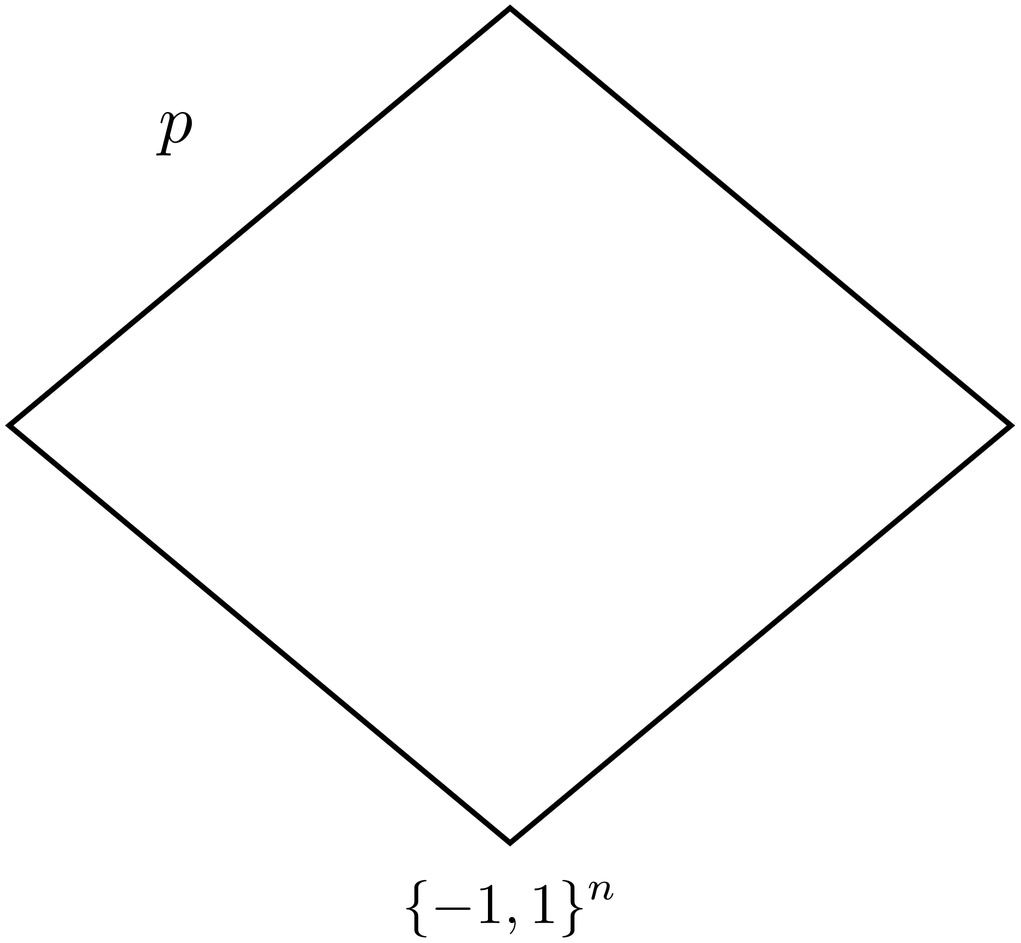

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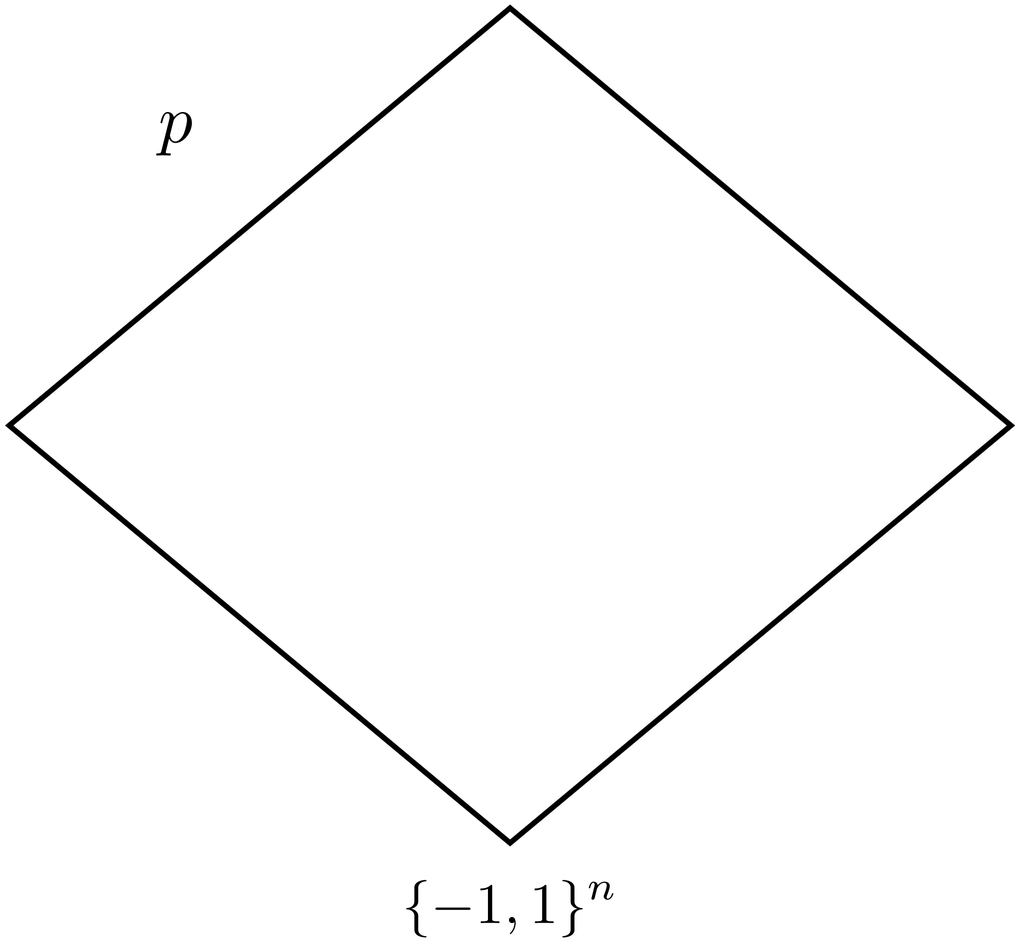

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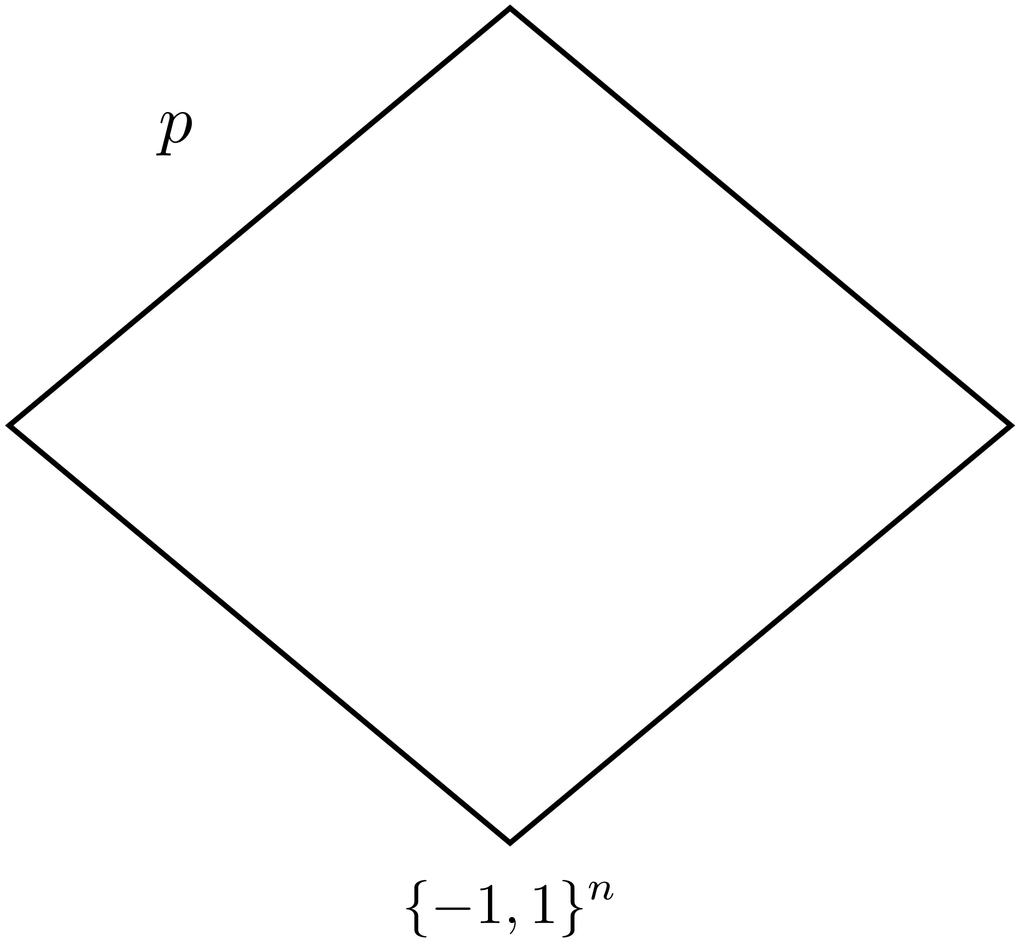

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Intuition of the Proof — 3 Conceptual Steps

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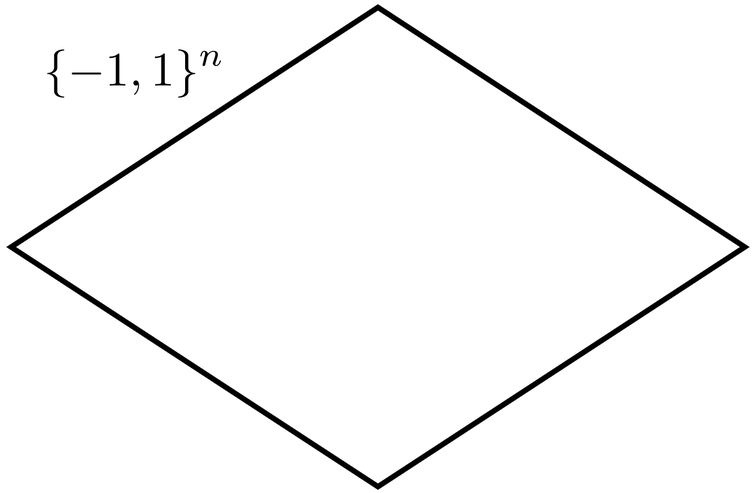
Intuition of the Proof — 3 Conceptual Steps

Isoperimetric Inequalities ⟷ **Symmetry in the Hypercube**



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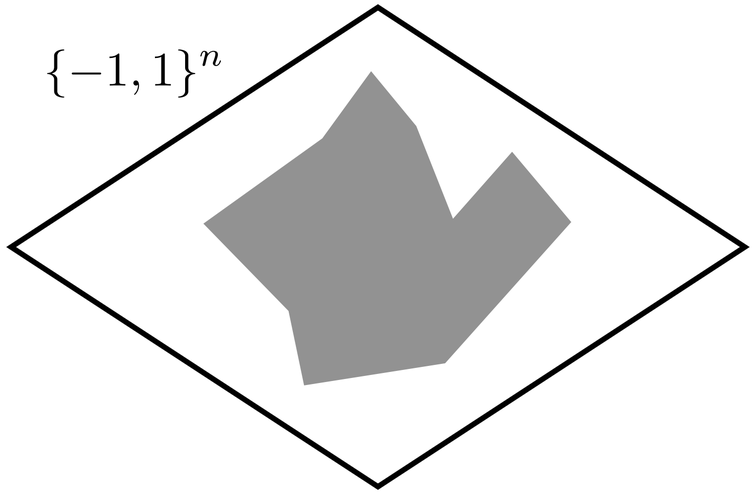

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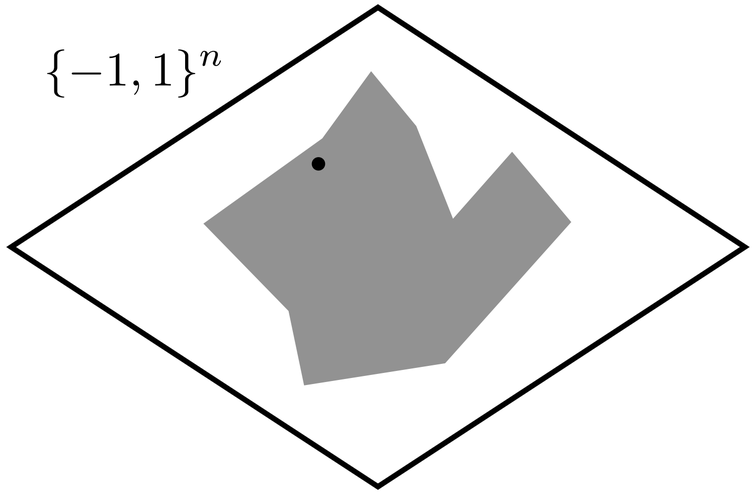

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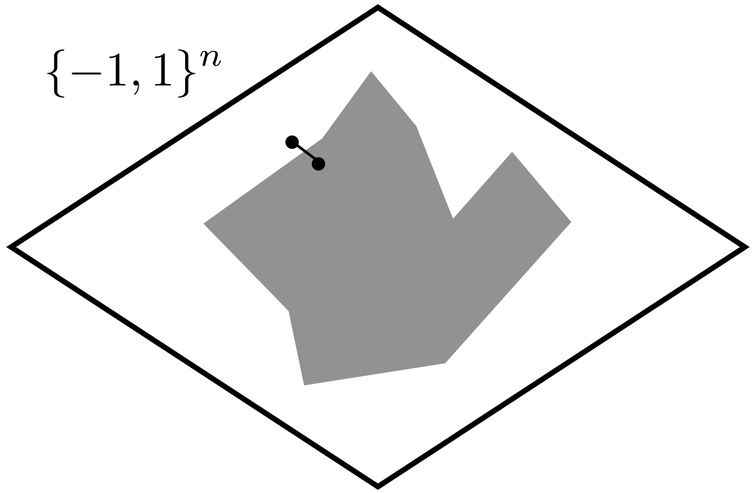

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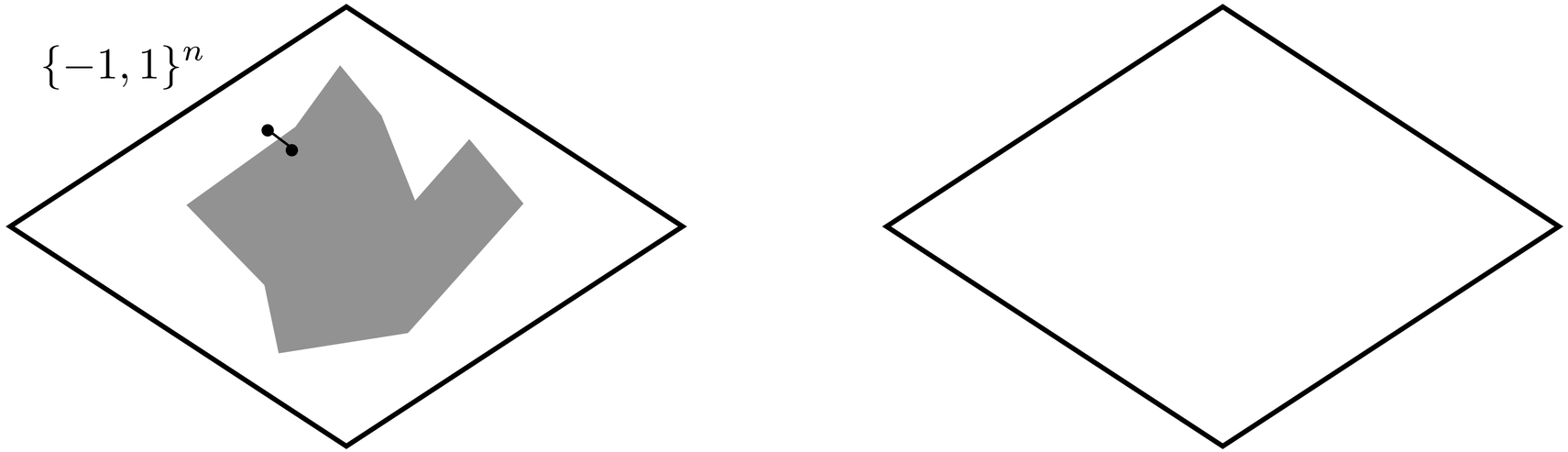

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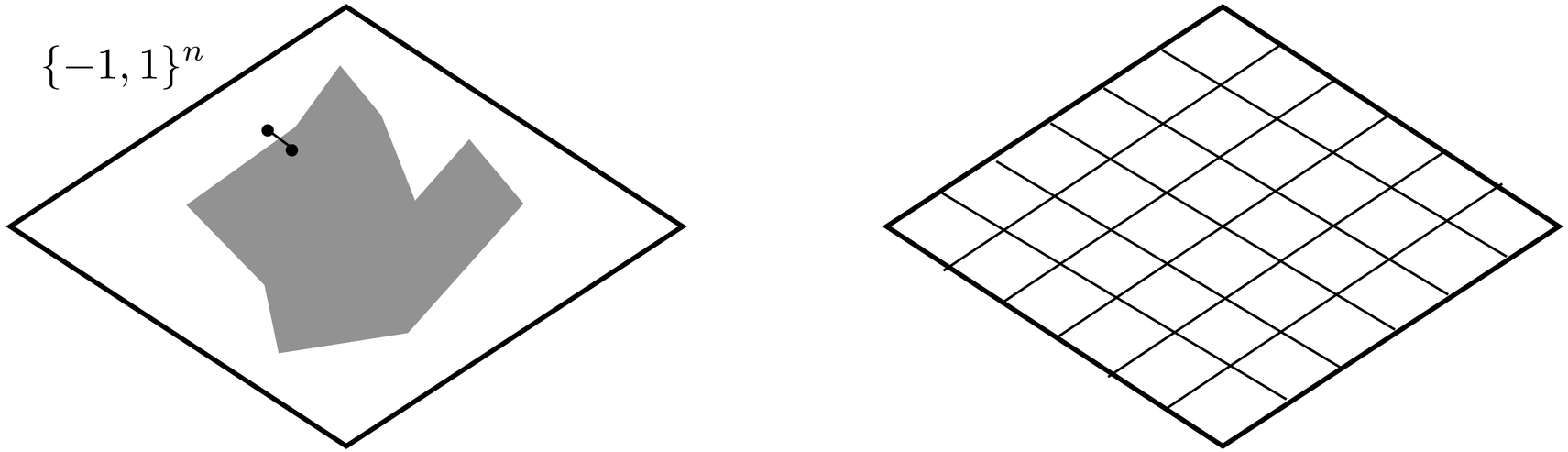

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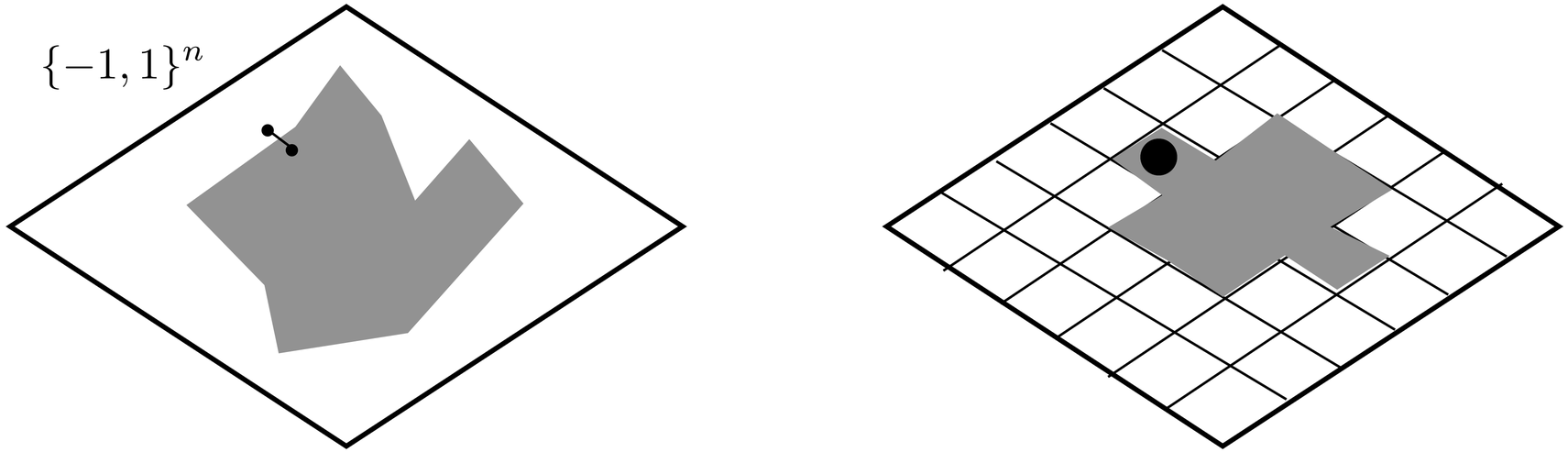

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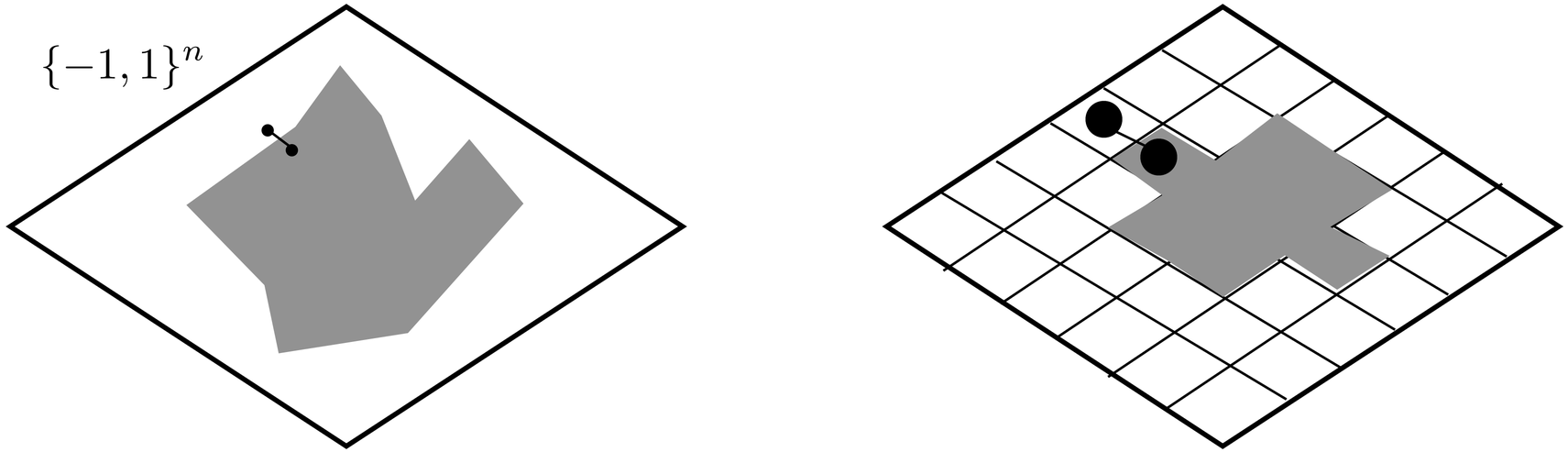

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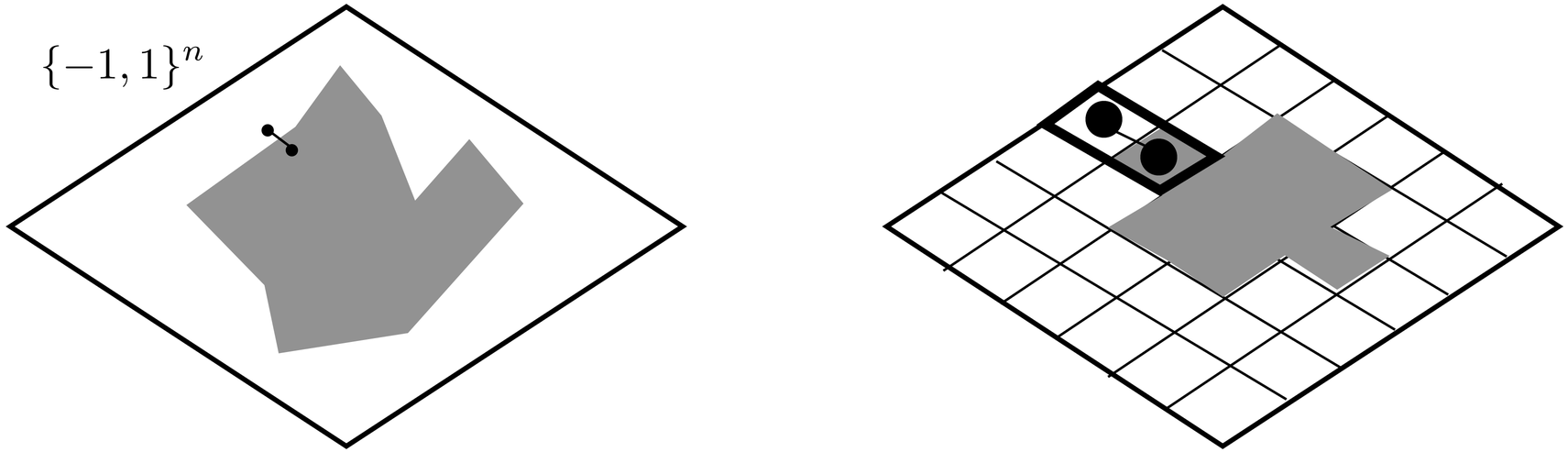

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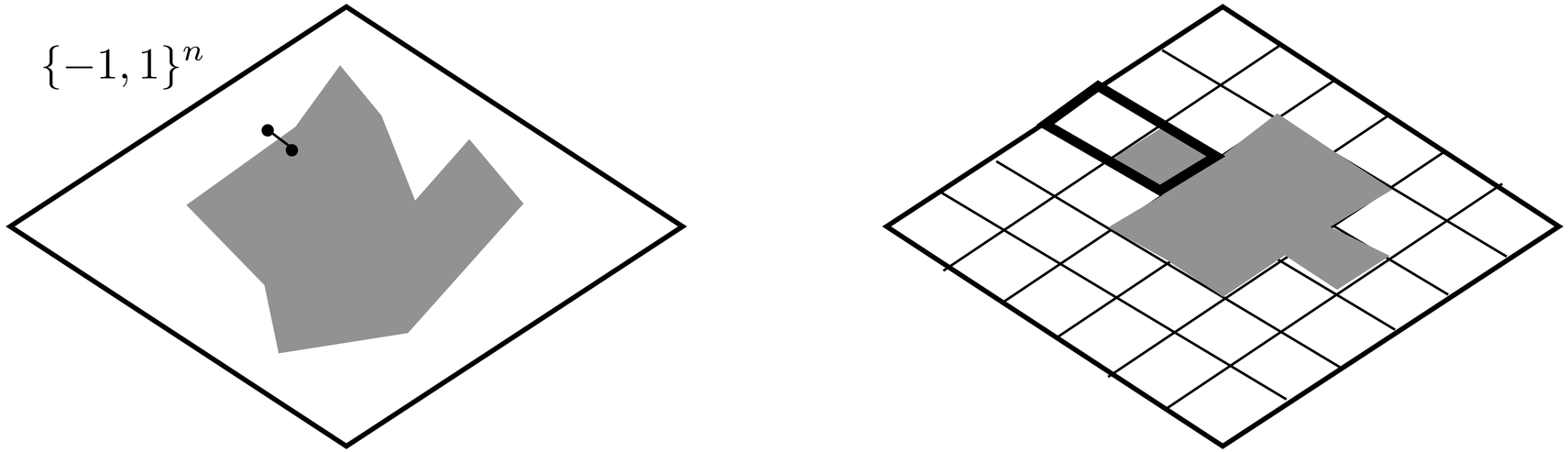

How to efficiently extract information from a distribution?

Question 1: Is it uniform/the distribution you are expecting?

Question 2: Are two distributions the same?

Question 3: Estimate properties of distributions …

Thanks!