18.937: TOPICS IN GEOMETRIC TOPOLOGY

Despite its various ramifications in symplectic topology and low-dimensional topology, Floer theory originated from Andreas Floer's attempt to resolve Arnol'd's conjecture concerning the number of fixed points of Hamiltonian diffeomorphisms. Now known as the Hamiltonian Floer theory, such a framework has inspired fundamental progress in the study of moduli spaces of pseudo-holomorphic maps and, more recently, algebraic topology of orbifolds. This course will cover foundational aspects of Hamiltonian Floer theory, from the beginning to the most recent advances in the field.

Instructor: Shaoyun Bai (shaoyunb@mit.edu). Office hours by appointment.

Time/Place: TR 9:30 am - 11:00 am, 66-144

Course website: https://math.mit.edu/~shaoyunb/18.937.html

Topics. (They may be subject to change during the semester.)

Basics of Hamiltonian Floer theory. [Sal99], [AD14], [MS04], [Par16]
- Motivations from Morse theory
- Conley–Zehnder indices, Fredholm operators on manifolds with cylindrical ends
- Transversality and compactness of moduli spaces of pseudo-holomorphic maps
- Gluing of solutions to Floer equations

Regularization of moduli spaces. [DK90], [AMS21, AMS23], [BX22a], [BX24]
- Implicit function theorems and Kuranishi models
- Global Kuranishi charts in Gromov–Witten theory
- Global Kuranishi charts in Hamiltonian Floer theory

Algebraic topology of orbifolds. [ALR07], [Par22], [Par21], [BX22b]
- Orbifolds, orbifold vector bundles, morphisms of orbifolds
- Orbifolds as global quotients by Lie groups
- Bordism theory of orbifolds
- Flow categories enriched by derived orbifolds
- Normally complex polynomial perturbations and refined invariants

Applications. [Sei15], [Wil20], [Sug21], [BX24]
- Quantum Steenrod operations and equivariant pants product
- Topology of Hamiltonian fibrations
- Hamiltonian group actions and periodic points

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[ALR07] Alejandro Adem, Johann Leida, and Yongbin Ruan, *Orbifolds and stringy topology*, Cambridge Tracts in Mathematics, vol. 171, Cambridge University Press, 2007.

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[DK90]   Simon Donaldson and Peter Kronheimer, *The geometry of four-manifolds*, Oxford Mathematical Monographs, Clarendon Press · Oxford, 1990.

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[Par22]   _____ , *Enough vector bundles on orbispaces*, Compos. Math. **158** (2022), no. 11, 2046–2081.

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[Sei15]   Paul Seidel, *The equivariant pair-of-pants product in fixed point Floer cohomology*, Geom. Funct. Anal. **25** (2015), no. 3, 942–1007. MR 3361776

[Sug21]   Yoshihiro Sugimoto, *On the Hofer–Zehnder conjecture for non-contractible periodic orbits in Hamiltonian dynamics*, https://arxiv.org/abs/2102.05273, 2021.

[Wil20]   Nicholas Wilkins, *A construction of the quantum Steenrod squares and their algebraic relations*, Geom. Topol. **24** (2020), no. 2, 885–970.