MATH 99R PROBLEM SET 11

Due at 9am on Thursday, December 3.

Throughout, let F be a number field.

(1) Let v be a nontrivial nonarchimedean norm on F, let $\chi_v : F_v^\times \to S^1$ be a continuous homomorphism, and let f_v be in $\mathcal{S}(F_v)$.
   (a) Prove that $Z_v(s, \chi_v, f_v)$ is a $\mathbb{C}$-rational function in q_v^{-s}.
   (Hint: break the integral into a sum over $\{x_v \in F_v^\times \mid v(x_v) = k\}$ for k in $\mathbb{Z}$, and use the fact that f_v is locally constant and compactly supported.)
   (b) Prove that $Z_v(s, \chi_v, f_v)/L(s, \chi_v)$ is entire.

(2) Let v be an archimedean norm on F such that $F_v \cong \mathbb{R}$. Suppose $\chi_v(x) = (x/|x|)^\varepsilon \|x\|_v^\nu$, where ε lies in $\{0, 1\}$ and ν lies in $i\mathbb{R}$.
   (a) Compute $\epsilon_v(s, \chi_v)$ when $\varepsilon = 0$.
   (Hint: use the test function $f_v(x) = e^{-\pi x^2}$.)
   (b) Let $f_v(x) = xe^{-\pi x^2}$. Show that $\widehat{f_v}(x) = ixe^{-\pi x^2}$.
   (c) Compute $\epsilon_v(s, \chi_v)$ when $\varepsilon = 1$.
   (Hint: use the test function $f_v(x) = xe^{-\pi x^2}$.)

(3) Let v be an archimedean norm on F such that $F_v \cong \mathbb{C}$. Suppose $\chi_v(z) = (z/|z|)^k \|z\|_v^\nu$, where k lies in $\mathbb{Z}$ and ν lies in $i\mathbb{R}$.
   (a) Let $f_v(z) = r^{|k|} e^{-2\pi k i\theta} e^{-2\pi r^2}$, where $z = re^{2\pi i\theta}$ denotes polar coordinates. Show that $\widehat{f_v}(z) = (ir)^{|k|} e^{2\pi k i\theta} e^{-2\pi r^2}$.
   (b) Compute $\epsilon_v(s, \chi_v)$.
   (Hint: use the test function $f_v(z) = r^{|k|} e^{-2\pi k i\theta} e^{-2\pi r^2}$.)