

## MATH 99R PROBLEM SET 10

Due at 9am on Thursday, November 19.

Throughout, let  $F$  be a number field.

- (1) Let  $f$  be in  $\mathcal{S}(\mathbb{A}_F)$ , and let  $x$  be in  $\mathbb{A}_F^\times$ . Prove that  $\sum_{\gamma \in F} f(x\gamma) = \|x\|^{-1} \sum_{\gamma \in F} \widehat{f}(x^{-1}\gamma)$ .
- (2) Let  $t$  be in  $\mathbb{R}_{>0}$ . Prove that  $\{x \in \mathbb{A}_F^\times \mid \|x\| = t\}$  has measure zero.  
(Hint: use  $\mathbb{A}_F^\times \cong \mathbb{A}_F^{\times,1} \times \mathbb{R}_{>0}$ .)
- (3) Let  $(I, S_0)$  be a modulus for  $F$ . Use the compactness of  $\mathbb{A}_F^{\times,1}/F^\times$  to prove that  $\mathcal{C}_{(I, S_0)}(F)$  is finite.