MATH 99R PROBLEM SET 8

Due at 9am on Thursday, November 5.

Problem (1) was taken from Dinakar Ramakrishnan and Robert Valenza's *Fourier Analysis on Number Fields*, and Problems (4) and (6) were taken from Daniel Bump's *Automorphic Forms and Representations*.

Throughout, let F be a number field.

(1) Let G be an abelian locally compact topological group, let m be a Haar measure on G, let H be a countable subgroup of G, and let K be a Borel subset of G such that $K + H = G$. If Y is a Borel subset of G such that $m(Y) > m(K)$, then there exist distinct y_1 and y_2 in Y such that $y_1 - y_2$ lies in H.

(2) Set $K = \{(x_v)_v \in \mathbb{A}_F \mid \|x_v\|_v \leq 1 \text{ for all } v \in M_F\}$. Prove that $K + F = \mathbb{A}_F$.
(Hint: emulate the case $F = \mathbb{Q}$ and use the product formula.)

(3) Let v be a nontrivial nonarchimedean norm on F. Prove that $\mathcal{O}_v$ is the closure of $\mathcal{O}_F$ in F_v.

(4) Let F_v be a nonarchimedean local field. Prove that every continuous group homomorphism $F_v^\times \to \mathbb{C}^\times$ is of the form $x \mapsto \chi(x)\|x\|_v^s$, where $\chi : F_v^\times \to S^1$ is a continuous group homomorphism, and s is a complex number.
(Hint: use Problem (1) from Problem Set 3.)

(5) Describe all the continuous group homomorphisms $\mathbb{R}^\times \to \mathbb{C}^\times$ as well as $\mathbb{C}^\times \to \mathbb{C}^\times$.
(Hint: note that $\mathbb{R}^\times = \{\pm 1\} \times \mathbb{R}_{>0}$ and $\mathbb{C}^\times = S^1 \times \mathbb{R}_{>0}$.)

(6) Prove that every continuous group homomorphism $\mathbb{A}_F^\times / F^\times \to \mathbb{C}^\times$ is of the form $x \mapsto \chi(x)\|x\|^s$, where $\chi : \mathbb{A}_F^\times / F^\times \to S^1$ is a continuous group homomorphism, and s is a complex number.
(Hint: use the compactness of $\mathbb{A}_F^{\times,1} / F^\times$.)