MATH 99R PROBLEM SET 7

Due at 9am on Thursday, October 29.

Problems (3)–(5) were taken from Dinakar Ramakrishnan and Robert Valenza's *Fourier Analysis on Number Fields*.

Throughout, let F be a number field.

(1) Let G be a locally compact topological group, and let m be a left Haar measure on G. Prove that, for any nonempty open subset $U \subseteq G$, we have $m(U) > 0$.

(2) Show that the usual topology on $\mathbb{A}_F^\times$ is strictly finer than the subspace topology from identifying it as the unit group of $\mathbb{A}_F$.

(3) Prove that the map $\mathbb{A}_F^\times \to \mathbb{A}_F^2$ given by $x \mapsto (x, x^{-1})$ is a homeomorphism onto its image.

(4) Show that $F^\times$ is discrete in $\mathbb{A}_F^\times$.

(5) Let x be in F. Prove that $\|x\|_v = 1$ for all v in M_F if and only if x is a root of unity.