(1) Let $A$ be a $2 \times 3$ matrix, $B$ be a $3 \times 4$ matrix, and $x$ a $3 \times 1$ vector. Which of the following multiplications make sense, and when they do, what are the dimensions of the resulting matrices?
   - $AA$
   - $AB$
   - $BA$
   - $Ax$
   - $Bx$
   - $xA$
   - $xx$

(2) Consider the system of equations
   \[
   \begin{align*}
   2x - y &= 4 \\
   4x - 2y &= a
   \end{align*}
   \]
   For what values of $a$ (if any) does the system have no solutions? What about infinitely many solutions? What about exactly one solution?

(3) Find the $LU$ factorisation of the following matrices, if it exists:
   \[
   \begin{pmatrix}
   1 & 2 \\
   1 & 3 \\
   \end{pmatrix}, \begin{pmatrix}
   1 & 2 & 1 \\
   2 & 5 & 0 \\
   0 & 0 & -1 \\
   \end{pmatrix}, \begin{pmatrix}
   0 & 1 \\
   1 & 0 \\
   \end{pmatrix}
   \]

(4) Convince yourself that the product of two upper-triangular matrices is again upper-triangular.

(5) Consider the following matrix equation:
   \[
   \begin{pmatrix}
   1 & 1 \\
   0 & 1 \\
   \end{pmatrix} \begin{pmatrix}
   1 & 0 \\
   1 & 1 \\
   \end{pmatrix} \begin{pmatrix}
   1 & 1 \\
   0 & 1 \\
   \end{pmatrix} \begin{pmatrix}
   x_1 \\
   x_2 \\
   \end{pmatrix} = \begin{pmatrix}
   1 \\
   0 \\
   \end{pmatrix}
   \]
   What are the values of $x_1$ and $x_2$? (Hint: the intended solution does not involve multiplying all the matrices together; instead use properties of upper/lower-triangular matrices.)

(6) In the playground at MIT, there are three freely exchanged currencies: fidget spinners, tide pods and Dogecoins. In a recent transaction, 200 fidget spinners and 1000 tide pods were exchanged for 150 fidget spinners, 750 tide pods and 62500 Dogecoins. In another transaction, 50 fidget spinners and 2000 tide pods were exchanged for 2500 tide pods and 25000 Dogecoins. At these exchange rates, if you were to purchase 10000 tide pods with 750 fidget spinners, how many Dogecoins would you get as change?