Recall the formula for compound interest: a principal amount of P invested at an annual interest rate r, compounded n times per year, will accrue to the amount

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

after t years. A principal amount of P invested at an annual interest rate r which is compounded *continuously* over t years will accrue to the amount

$$A(t) = P\mathrm{e}^{rt},$$

where $\mathrm{e} = 2.718281828\ldots$ is the base of the natural logarithm.

1. One hundred grams of a particular radioactive substance decays according to the function $M(t) = 100\mathrm{e}^{-t/650}$, where $t > 0$ measures time in years. When does the mass reach 50 grams?

2. A culture of bacteria has a population of 150 cells when it is first observed. The population doubles every 12 hours, which means that its population is governed by the function $p(t) = 150 \cdot 2^{t/12}$, where t is the number of hours after the first observation.

   a. Verify that $p(0) = 150$, as claimed.
   b. Show that the population doubles every 12 hours, as claimed.
   c. What is the population 4 days after the first observation?
   d. How long does it take for the population to triple in size?
   e. How long does it take for the population to reach 10,000 cells?

3. The half-life of an antibiotic in the bloodstream is 10 hours. If an initial dose of 20 milligrams is administered, the quantity left after t hours is modeled by $Q(t) = 20\mathrm{e}^{-0.0693t}$, for $t \geq 0$.

   a. Find the instantaneous rate of change of the amount of antibiotic in the bloodstream, for $t \geq 0$.
   b. How fast is the amount of antibiotic changing at $t = 0$? at $t = 2$?
   c. Evaluate and interpret $\lim_{t\to\infty} Q(t)$ and $\lim_{t\to\infty} Q'(t)$.

4. A \$200 investment in a savings account grows according to $A(t) = 200\mathrm{e}^{0.0398t}$, for $t \geq 0$, where t is measured in years.

   a. Find the balance of the account after 10 years.
   b. How fast is the account growing (in dollars per year) at $t = 10$?
   c. Use your answers to parts a. and b. to write the equation of the line tangent to the curve $A(t) = 200\mathrm{e}^{0.0398t}$ at the point $(10, A(10))$.

5. Iodine-123 is a radioactive isotope used in medicine to test the function of the thyroid gland. If a 350 microcurie (μCi) dose of iodine-123 is administered to a patient, the quantity $Q = Q(t)$ left in the body after t hours is approximately $Q(t) = 350\left(\frac{1}{2}\right)^{t/13.1}$.

   a. How long does it take for the level of iodine-123 to drop to 10μCi?
   b. Find the rate of change of the quantity of iodine-123 at 12 hours, at 1 day, and at 2 days. What do your answers say about the rate at which iodine decreases as time increases?

6. *(Banker's Rule of 70).* If you earn interest at a rate of $R\%$, continuously compounded, your money doubles after approximately $70/R$ years. For example, if $R = 5\%$ then you will double your money after $70/5 = 14$ years. Explain why.

7. The U.S. government reports the rate of inflation (as measured by the Consumer Price Index) both monthly and annually. Suppose that for a particular month, the *monthly* rate of inflation is reported as 0.8%. Assuming that this rate remains constant, what is the corresponding *annual* rate of inflation? Is the annual rate 12 times the monthly rate? Explain.

WeBWorK-style questions. The following optional questions will be asked in a style similar to questions present in midterm 1 and more generally on WeBWorK.

8. Assume the annual inflation rate is r. This means that X dollars t-years from now is worth Xe^{-rt} dollars in present times. We want to know if it's better to receive 2000 dollars now or 2200 in two years. Find r so that both options are equivalent.

$$r = \qquad .$$

9. I invest P dollars compounded continuously at a rate r. My bank tells me it will make me a millionaire in 10 years, and a billionaire in 50 years. Find P and r.

$$P = \qquad , r = \qquad .$$

10. Bob invests 1000 dollars compounded annually at a rate of 20%. Ross only invests 800 dollars, at the same rate, but compounded continuously. After t years, Bob and Ross will have the exact same amount of money in the bank. Find t.

$$t = \qquad \text{years.}$$