Recall the definition of *elasticity*, namely,

$$\varepsilon(p) = \frac{\mathrm{d}q}{\mathrm{d}p} \cdot \frac{p}{q} = \frac{\frac{\mathrm{d}q}{q}}{\frac{\mathrm{d}p}{p}} \approx \frac{\%\text{ change in demand}}{\%\text{ change in price}},$$

where $q = q(p)$ expresses demand as a function of price p; this quantity is related to variations of the revenue via $\frac{\mathrm{d}R}{\mathrm{d}p} = q(1 + \varepsilon(p))$.

If $-\infty < \varepsilon < -1$, the demand is said to be *elastic*, while if $-1 < \varepsilon < 0$, the demand is said to be *inelastic*. If $\varepsilon = -1$ then the product is said to be *unit-elastic*.

1. (*) Based on sales data over the past year, the owner of a DVD store devises the demand function $q(p) = 40 - 2p$, where $q(p)$ is the number of DVDs that can be sold in one day at a price of p dollars.

   a. According to the model, how many DVDs can be sold in a day at a price of \$10?

   b. According to the model, what is the maximum price that can be charged (above which no DVDs can be sold)?

   c. Find the elasticity function for this demand function.

   d. For what prices is the demand elastic? Inelastic? Unit-elastic?

   e. If the price of DVDs is raised from \$10.00 to \$10.25, what is the exact percentage decrease in demand (using the demand function)?

   f. If the price of DVDs is raised from \$10.00 to \$10.25, what is the approximate percentage decrease in demand (using the elasticity function)?

2. Compute the elasticity for the exponential demand function $q(p) = a\mathrm{e}^{-bp}$, where a and b are positive real numbers. For what prices is the demand elastic? Inelastic?

3. Show that the demand function $q(p) = a/p^b$, where a and b are positive real numbers, has a constant elasticity for all positive prices.

4. For each of the following equations, (i) use implicit differentiation to find $\frac{\mathrm{d}y}{\mathrm{d}x}$, and (ii) find the slope of the tangent line to the curve at the given point.

   a. $x^4 + y^4 = 2$ at $(-1, 1)$.

   b. $x = \mathrm{e}^y$ at $(2, \ln 2)$.

   c. $y^2 = 4x$ at $(1, 2)$.

5. Use implicit differentiation to find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

   a. $\sin xy = x + y$.

   b. $\mathrm{e}^{xy} = 2y$.

   c. $x + y = \cos y$.

6. Use implicit differentiation to find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

   a. $x + y^2 = 1$.

b. $2x^2 + y^2 = 4$.

c. $x + y = \sin y$.

7. a. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, where $\sqrt{3x^7 + y^2} = \sin^2 y + 100xy$.

   b. Find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$, where $\sqrt{y} + xy = 1$.

8. a. Find where the tangent line of $y^2 = x^3 - 3x + 1$ is horizontal. This curve is called an *elliptic curve*.

   b. The "Limaçon" curve is $(x^2 + y^2 - 4x)^2 = 2(x^2 + y^2)$. Find the equation of tangent lines at the points where $x = 1$.

9. Use logarithmic differentiation to compute the derivative of the following functions on their domain

   a. x^x

   b. x^{e^x}

   c. $\sqrt{\dfrac{\cos(x)e^x}{x^2(x-1)^6}}$.

   d. Use logarithmic differentiation to prove the power, product, and quotient rules.