Questions marked with $(*)$ are more involved than the other questions.

1. The company The Dark Side of the Spoon is selling high end sporks. Their monthly revenue function is $R(q) = \dfrac{\left(16\sqrt{q}+q\right)}{q+1}$. At their current price, the company is selling 4 sporks a month. Using the marginal revenue $\dfrac{\mathrm{d}R}{\mathrm{d}q}$, determine whether a small increase in price would lead to an increase or decrease of the monthly revenue.

2. Suppose the position of an object moving horizontally after t seconds is given by the function $s = f(t) = t^2 - 4t$ for $0 \le t \le 5$, where s is measured in metres, with $s > 0$ corresponding to positions to the right of the origin.

   a. Graph the position function.
   b. Find and graph the velocity function. When is the object stationary, moving to the right, and moving to the left?
   c. Determine the velocity and acceleration of the object at $t = 1$.
   d. Determine the acceleration of the object when its velocity is zero.
   e. On what intervals is the speed increasing?

3. Economists use *production functions* to describe how the output of a system varies with respect to another variable such as labour or capital. For example, the production function $P(L) = 200L + 10L^2 - L^3$ gives the output of a system as a function of the number of labourers L. The *average product* $A(L)$ is the average output per labourer when L labourers are working; that is, $A(L) = P(L)/L$. The *marginal product* $M(L)$ is the approximate change in output when one additional labourer is added to L labourers; that is, $M(L) = \dfrac{\mathrm{d}P}{\mathrm{d}L}$.

   a. For the given production function, compute and graph P, A, and M.
   b. Suppose the peak of the average product curve occurs at $L = L_0$, so that $A'(L_0) = 0$. Show that for a general production function, $M(L_0) = A(L_0)$.

4. A store manager estimates that the demand for an energy drink decreases with increasing price according to the function $d(p) = \dfrac{100}{p^2+1}$, which means that at price p (in dollars), $d(p)$ units can be sold. The revenue generated at price p is $R(p) = p \cdot d(p)$, i.e. price multiplied number of units.

   a. Find and graph the revenue function.
   b. Find and graph the marginal revenue $R'(p)$.
   c. From the graphs of R and R', estimate the price that should be charged to maximize the revenue.

5. For parts a. and b., identify the inner and outer functions in the composition (i.e. choose functions $f(x)$ and $g(x)$ so that the given function equals $f(g(x))$.) For part c., express the given function as the composition of three functions $f(g(h(x)))$.

   a. $A(x) = \cos^4 x$.
   b. $B(x) = (x^2 + 10)^{-5}$.

c. $C(x) = \cos^4(x^2 + 1)$.

6. Two composite functions are given that look similar, but in fact are quite different. Identify the inner function $u = g(x)$ and the outer function $y = f(u)$, then evaluate $\frac{\mathrm{d}y}{\mathrm{d}x}$ using the chain rule.

   a. $y_1 = \cos^3 x$; $y_2 = \cos x^3$.

   b. $y_1 = (\mathrm{e}^x)^3$; $y_2 = \mathrm{e}^{(x^3)}$.

7. Let $h(x) = f(g(x))$ and $k(x) = g(g(x))$. Use the table to compute the following derivatives.

a. $h'(1)$	b. $h'(2)$
c. $h'(3)$	d. $k'(3)$
e. $k'(1)$	f. $k'(5)$

x	1	2	3	4	5
$f'(x)$	-6	-3	8	7	2
$g(x)$	4	1	5	2	3
$g'(x)$	9	7	3	-1	-5

8. (*) Calculate the derivative of the following functions.

   a. $y = \cos^4(7x^3)$.

   b. $y = \tan\left(\mathrm{e}^{\sqrt{3x}}\right)$.

   c. $y = (1 - \mathrm{e}^{-0.05x})^{-1}$.

9. (*) Assume that f and g are differentiable on their domains with $h(x) = f(g(x))$. Suppose the equation of the line tangent to the graph of g at the point $(4, 7)$ is $y = 3x - 5$ and the equation of the line tangent to the graph of f at $(7, 9)$ is $y = -2x + 23$.

   a. Calculate $h(4)$ and $h'(4)$.

   b. Determine an equation of the line tangent to the graph of h at the point on the graph where $x = 4$.

10. (*) Let $f(x) = \cos^2 x + \sin^2 x$.

   a. Use the chain rule to show that $f'(x) = 0$.

   b. Assume that if $f' = 0$, then f is a constant function. Calculate $f(0)$ and use it with part a. to explain why $\cos^2 x + \sin^2 x = 1$.