Questions marked with $(*)$ are more involved than the other questions. Questions with multiple stars are harder and should only be attempted after the rest was completed.

1. Let $f(x) = \sqrt{3x+1}$ and let $a = 8$.

   a. Use the definition of the derivative to find $f'(a)$.

   b. Determine the equation of the line tangent to the graph of f at the point $(a, f(a))$.

2. Find the derivative of the following functions.

   a. $g(x) = 6x^5 - x$.

   b. $f(t) = 6\sqrt{t} - 4t^3 + 9$.

   c. $s(t) = 4\sqrt{t} - \frac{1}{4}t^4 + t + 1$.

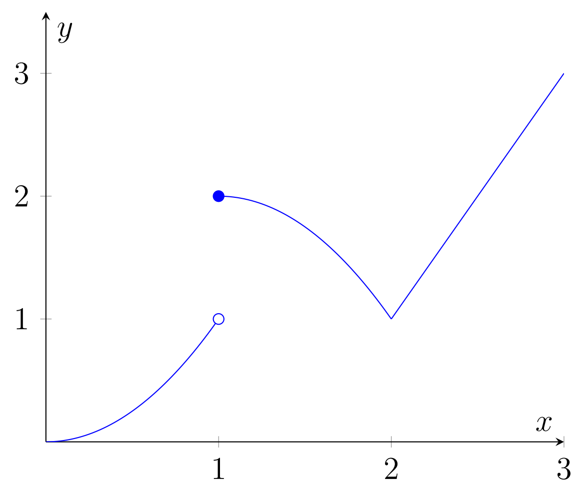
3. Consider the curve $y = x^3 - 4x^2 + 2x - 1$. Find the equation of the line tangent to the curve at $x = 2$.

4. $(*)$ Use the graph of f in the figure to do the following.

   a. Find the values of x in $(0, 3)$ at which f is not continuous.

   b. Find the values of x in $(0, 3)$ at which f is not differentiable.

   c. Sketch a graph of f'.



5. Let

$$f(x) = \begin{cases} 2x^2 & \text{if } x \leq 1, \\ ax - 2 & \text{if } x > 1. \end{cases}$$

   Determine the value (if it exists) of a for which f' is continuous at $x = 1$.

6. Let $f(x) = 2x^3 - 3x^2 - 12x + 4$.

   a. Find all points on the graph of f at which the tangent line is horizontal.

   b. Find all points on the graph of f at which the tangent line has slope 60.

7. $(*)$ The limit $\lim_{h \to 0} \dfrac{(1+h)^8 + (1+h)^3 - 2}{h}$ represents $f'(a)$ for some function f and some real number a.

a. Find a possible function f and number a.

b. Evaluate the limit by computing $f'(a)$.

c. Do the same to compute $\lim_{x\to 1} \dfrac{\ln(x)}{x-1}$.

d. *(Final 2016)* Do the same to compute $\lim_{x\to\frac{\pi}{4}} \dfrac{\sin(x)-\cos(x)}{x-\frac{\pi}{4}}$.

8. ($*$) *(Final 2010)* If a function $y = f(x)$ is differentiable and $x = 3$ and $f'(3) = 5$, find the limit $\lim_{x\to 3} \dfrac{x^2-3x}{f(3)-f(x)}$.

9. ($**$)

   a. Let $f(x), g(x)$ be function and fix some a. Assume $f(a) = g(a) = 0$, and $g'(a) \neq 0$. Observe that in the spirit of the previous questions, we can compute

   $$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{x-a}{g(x)-g(a)} \frac{f(x)-f(a)}{x-a} = \frac{f'(a)}{g'(a)}.$$

   b. Use this rule to compute $\lim_{x\to 0} \frac{e^x-e^{-x}}{e^x-1}$.

10. Find the derivative of the following functions.

    a. $g(x) = 6x - 2x\mathrm{e}^x$.

    b. $g(w) = \mathrm{e}^w(5w^2+3w+1)$.

    c. $f(x) = \left(1+\frac{1}{x^2}\right)(x^2+1)$.

11. ($*$) Compute the derivative of the following functions.

    a. $h(x) = \dfrac{x\mathrm{e}^x}{x+1}$.

    b. $h(x) = \dfrac{x+1}{x^2\mathrm{e}^x}$.

12. ($*$) Assuming that the first and second derivatives of f and g exist at x, find a formula for $\dfrac{\mathrm{d}^2}{\mathrm{d}x^2}\big(f(x)g(x)\big)$.

13. ($**$) Consider the function $f(x) = \dfrac{x}{1-x}$. If $a \neq 1$, define $g(a)$ to be the x-intercept of the tangent line of $y = f(x)$ at $x = a$. Find a formula $g(a)$.

14. ($**$)

    a. Show that the tangent lines to the x-intercepts of the polynomial $2x^2+3x+1$ are perpendicular.

    b. Find a condition on a, b, c so that the two tangent lines of $y = ax^2+bx+c$ at the x-intercepts are perpendicular.

15. ($**$) Suppose the function $f(x)$ is twice differentiable and $f(x^2) = f(x) + x^2$. Find $f'(1)$ and $f''(1)$.

16. $(***)$ Suppose $a > 0$. Consider the function $f(x) = \dfrac{1}{x}$. Determine $A(a)$, defined as the area of the triangle $\mathcal{T}$ formed between the tangent line of $y = f(x)$ at $x = a$, the x-axis, and the y-axis. Drawing the situation can be really useful.