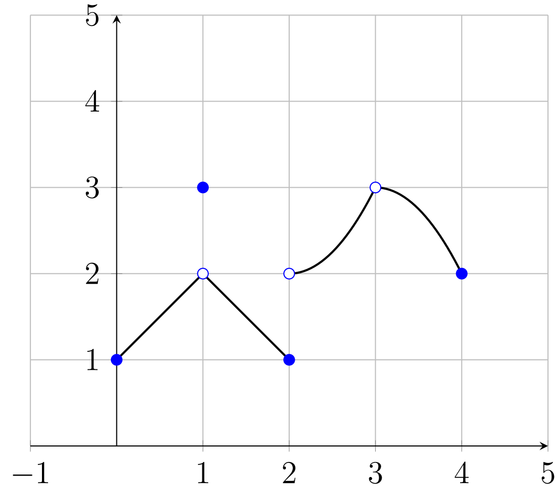
1. Determine the points at which the following function f has discontinuities. At each point of discontinuity, state the conditions in the continuity checklist that are violated.



2. Compute $\lim_{x\to 4} \dfrac{x+\sqrt{x}-6}{x-4}$.

3. If $\lim_{x\to 1} g(x) = 6$ and $\lim_{x\to 1} \dfrac{g(x)}{f(x)-1} = 3$, find $\lim_{x\to 1} f(x)$.

4. Consider three functions $f(x), g(x), h(x)$ such that

$$\lim_{x\to 1} f(x) = 2,\ \lim_{x\to 1} g(x) = -2,\ \lim_{x\to 1} h(x) = 2,$$

$$\lim_{x\to 2} f(x) = 6,\ \lim_{x\to 2} g(x) = 3,\ \lim_{x\to 2} h(x) = 2,$$

$$\lim_{x\to 3} f(x) = -1,\ \lim_{x\to 3} g(x) = -1,\ \lim_{x\to 3} h(x) = 1.$$

Compute $\lim_{x\to 1} \dfrac{\sqrt{f(h(f(x))) + g(x)}}{2g(x) + f(x)h(x) + x}$.

5. Let

$$f(x) = \begin{cases} \frac{x^2-4x+3}{x-3} & \text{if } x \neq 3, \\ 2 & \text{if } x = 3. \end{cases}$$

Determine whether or not $f(x)$ is continuous at $a = 3$. Use the continuity checklist to justify your answer.

6. Determine the interval(s) on which the function $f(x) = \dfrac{x^5+6x+17}{x^2-9}$ is continuous.

7. Let

$$f(x) = \begin{cases} 2x & \text{if } x < 1, \\ x^2+3x & \text{if } x \geq 1. \end{cases}$$

   a. Use the continuity checklist to show that f is not continuous at 1.
   b. Is f left-continuous or right-continuous at 1?
   c. State the interval(s) of continuity.

8. Find an interval of length 1 containing a number x such that $2^x + 3^x = 4^x$.

9. You are shopping for a \$150,000, 30-year loan to buy a house. The monthly payment is

$$m(r) = \frac{150000(r/12)}{1 - (1 + r/12)^{-360}},$$

where r is the annual interest rate. Suppose banks are currently offering interest rates between 6% and 8%.

   a. Use the intermediate value theorem to show that there is a value of r in the interval $(0.06, 0.08)$ – i.e., an interest rate between 6% and 8% – that allows you to make monthly payments of \$1000 per month. You can use an online calculator for numerical computations.

   b. Use a graph to illustrate your explanation to part a. Then determine the interest rate you need for monthly payments of \$1000.

10. Determine the value of the constant a for which the function

$$f(x) = \begin{cases} \frac{x^2+3x+2}{x+1} & \text{if } x \neq -1, \\ a & \text{if } x = -1. \end{cases}$$

is continuous at -1.

11. Let

$$f(x) = \begin{cases} x^2 + x & \text{if } x < 1, \\ a & \text{if } x = 1, \\ 3x + 5 & \text{if } x > 1. \end{cases}$$

   a. Determine the value of a for which f is left-continuous at 1.

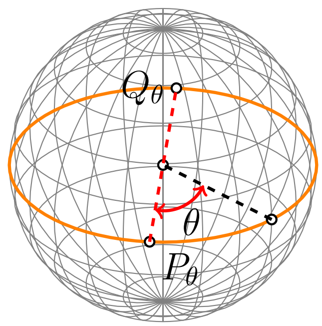
   b. Determine the value of a for which f is right-continuous at 1.

   c. Is there a value of a for which f is continuous at 1? Explain why or why not.

12. **(*)** *This is a bonus question, only attempt it if you have finished the rest.*
The goal of this question is to prove the following: On the equator, there are always two points diametrically opposed (*i.e.* symmetric with respect to the center of the earth) with the same temperature. Define a function as follows

$$f(\theta) = \text{temperature at } P_\theta - \text{temperature at } Q_\theta,$$

where θ is an angle between $0°$ and $360°$, and P_θ and Q_θ are in the following picture.



Use the IVT to show that there is an angle x between $0°$ and $180°$ such that $f(x) = 0$. What does this show?

Remark. This is nothing special about the equator, on any longitude or lattitude there are always two points opposed with the same temperature.